THE OPEN UNIVERSITY
A SCIENCE FOUNDATION COURSE

UNIT 10
MODELLING THE BEHAVIOUR OF LIGHT

WAVE–PARTICLE DUALITY

STUDY GUIDE

This Unit has four components: the text, an experiment, an AV sequence, and the TV programme 'Light—in search of a model'.

The experiment has four parts, each designed to be very quick and easy to perform. Their aim is to let you see for yourself what optical diffraction patterns look like, so that you will have first-hand knowledge of the effects being discussed later in the text. The experiment is interspersed with ITQs in such a way that the answers to the ITQs provide sequential comments on parts of the experiment. You should find these comments helpful if you are having any difficulties with the experiment.

In order to perform the experiment you will need to purchase a clear-glass, single-wire-filament light bulb. A low-power (25 watt) candle bulb is ideal. If you have not already done so, you should buy this bulb now.

The AV sequence in Section 3.1 introduces you to some of the nomenclature and mathematics used to describe waves and wave motion. It forms an important and integral part of the story of this Unit, and you should work through it at the indicated place in the text. It should take you about an hour to work through the sequence.

The TV programme, 'Light—in search of a model', shows a number of experiments and demonstrations that should help you to visualize and understand both the wave *and* particle aspects of the 'dual model' of light discussed in the text. Ideally, you should have at least browsed through all the text before watching the programme; however, this is not essential. You will find notes for the TV programme in Section 8.

1 INTRODUCTION

MODEL An artificial construction invented to represent or to simulate the properties, the behaviour, or the relationship between individual parts of the real entity being studied. (Unit 1)

This Unit is about modelling the properties and behaviour of light—a field of knowledge that we believe today to be one of the best understood areas in the whole of physics. That's quite a claim. But, as you will see during your study of the Unit, the position has not always been so clear cut. Furthermore, you may well begin to feel during your studies that the account we give here is, to say the least, not consistent with common sense—requiring, as it does, two different models to explain all the observations.

Physics attempts to provide an understanding of physical phenomena on a *universal* scale: from the farthest galaxies to the smallest domain of the most fundamental particles of matter, and from objects at rest to objects travelling close to the speed of light. Your common sense, however, is a product of your very limited everyday experience—a much smaller reservoir from which to draw. The ultimate test of the usefulness of any model or theory is its ability to fit in with the results of experiments, *not* its ability to fit in with common sense. So, if this dual model of light that we present you with in this Unit appears to defy common sense, then it is your reliance on common sense that must be jettisoned—because the dual model does fit extremely well with experiment.

So how was this dual model arrived at? Well, to answer this question, we first have to go back a bit in history. Optics is a very old science: Isaac Newton published his *Treatise on Opticks* in 1704; and one of the earliest scientific models of light had been put forward by a contemporary of Newton's, Christiaan Huygens (1629–1695). As a result of the work of one of the greatest physicists of the 19th century, James Clerk Maxwell (1831–1879), it was thought that light could be understood as a 'wave phenomenon'—in particular as *electromagnetic* waves which, as we shall show later in the Unit, is a term that can be applied not just to light but to a whole spectrum of radiation, ranging from radio waves, microwaves and infrared radiation, through to ultraviolet radiation, X-rays and gamma radiation.

This wave model is still perfectly satisfactory today for explaining the vast majority of the properties and behaviour patterns of light (that is why the greater part of this Unit is devoted to it). However, at the beginning of the 20th century it was realized by physicists—and notably by Albert Einstein—that there were some phenomena for which the wave model seemed incapable of providing a satisfactory explanation. Two such phenomena, which we shall be looking at in more detail later, concerned the photoelectric and Compton effects, which each involve the interaction of radiation with matter. The results of these experiments could be explained satisfactorily by modelling light not as a wave motion, but rather as *a stream of particles.* Over the years, this particle model has been extensively developed—but not at the expense of the wave model, rather as complementary to it.

We now believe that, in order to understand the behaviour of light from a macroscopic point of view, *both* of these models are required—a *duality* of models is needed. Even though it may seem very strange at first, you're just going to have to get used to this idea of **wave–particle duality**! In scientific terms, it does make very good sense.

REFLECTION OF A WAVE

REFRACTION OF A WAVE

2 WHY A WAVE MODEL ANYWAY?

The aim of the first part of this Unit is to introduce you to, and explain the details of, the wave model of the behaviour of light. But before embarking on that task, there is a fundamental question that we should address: What evidence is there for giving any credence whatsoever to a wave model of light? Perhaps the easiest way to answer this question is to compare the behaviour of rather better known wave motions. In particular, we can compare the behaviour of light with the behaviour of seismic waves (Units 5–6), with the more familiar water waves, or even with sound waves. So how does light compare with these wave motions?

2.1 WAVE PHENOMENA

Let's look first of all at the ability of a wave to *transfer energy* from one place to another. Clearly there is no doubt that seismic waves carry huge amounts of energy; and if you've ever been surfing or swimming in a choppy sea, you'll not need much convincing that water waves too can carry large amounts of energy. That sound waves transfer energy is perhaps not so obvious, though even here, with a little pause for thought, you can probably think of examples in which energy is transferred by sound. Perhaps you've felt in your stomach the very-low-frequency sound sometimes put out at discos; or what about the ability of very loud music to make objects in your room vibrate or rattle? You've probably also heard that the human voice can shatter a crystal glass; although this is by no means as easy to achieve as you may have been led to believe, it is definitely possible.

So, is light also capable of transferring energy? Well, yes—as you saw in Unit 9, the energy we use on Earth is all ultimately derived from the Sun. It's solar energy that is used in photosynthesis, which in turn enables plants to grow. It is from plants that we derive our own energy, either by eating the plants themselves or by eating animals that have ultimately derived their own energy stores from plants. It is also from plants that the Earth's stocks of fossil fuels have been derived over the past millions of years—fuels that we now use to power our modern technological society. Nowadays, we can convert the energy carried by light even more directly—we can use solar cells, and related devices, to convert the energy of light directly into electricity.

So it appears that light most definitely stands comparison with other wave motions on this first count. Waves in general are able to transfer energy from one place to another—and so can light.

What other properties do waves in general possess? As you are aware from your study of seismic waves, waves can be **reflected** (Units 5–6). Sound waves, too, can be reflected—think of the echo you can produce when shouting into a rocky canyon. And as you can see from Figure 1, it is also possible to 'bounce' water waves (in this case, ripples on the surface of a tank of water) off hard surfaces. So, is light similarly reflected? The answer is, of course, yes. As Figure 2 shows, light waves are reflected by mirror surfaces in such a way that the angle of incidence i is equal to the angle of reflection R (Units 5–6). So, once again, light does seem to behave like a wave.

Another property of waves is their ability to be **refracted**. Refraction is the process in which a wave's direction of propagation is changed as its speed changes. You saw in Units 5–6 how seismic waves could be bent by this process of refraction. It's also true that sound waves can be refracted. For instance, you have perhaps noticed that sound appears to travel more easily in the evening over water. As Figure 3 illustrates, this is a refraction phenomenon, which occurs because the air above the water in the evening is at different temperatures, and thus at different densities, at different heights

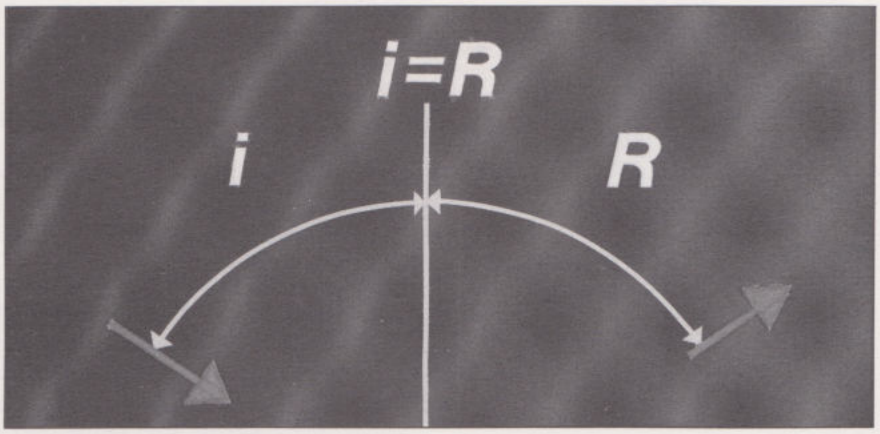


FIGURE 1 The reflection of water waves. The waves are produced by a paddle (off the left-hand side of the photograph) moving vertically up and down in the water in a tank. When the tank is illuminated, and viewed from above, the crests (maxima) of the waves show up as dark lines, the troughs (minima) as bright lines. When an obstacle is placed in the tank, the waves are reflected by it, as shown by the arrows.

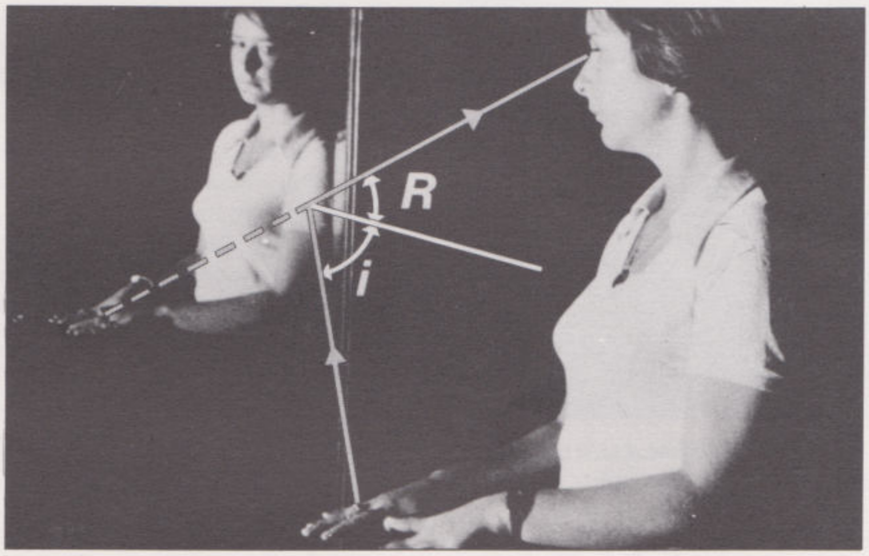


FIGURE 2 A mirror reflects the light waves from an object towards the onlooker's eye in such a way that the angle of incidence *i* of the waves as they strike the mirror is equal to the angle of reflection *R*. The image of the object appears to be as far behind the mirror as the object is in front of it.

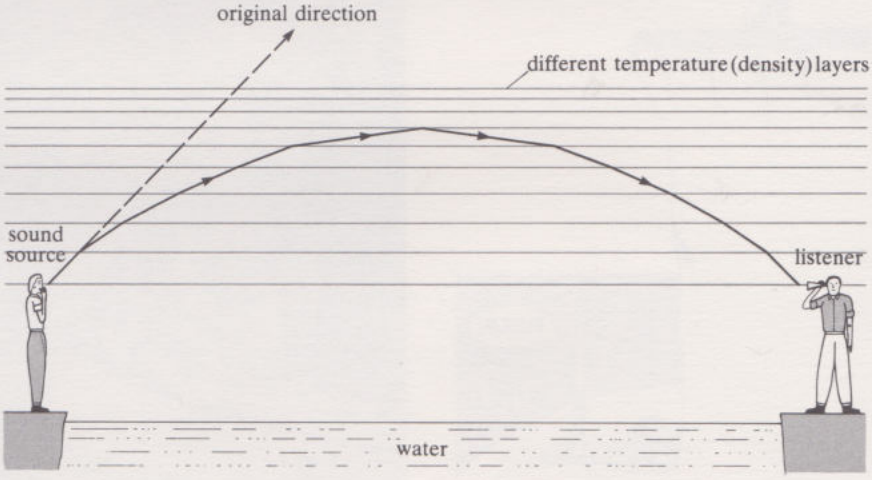


FIGURE 3 The layers of air are at different temperatures and have different densities. This causes the sound waves to change direction at the boundary of each layer. Sound that would have been directed well away from the listener, will now be 'bent back' again to the ground.

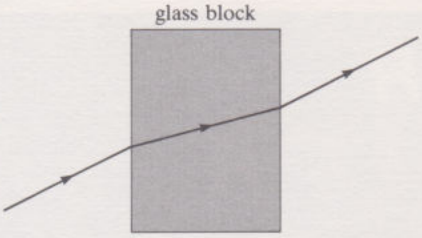


FIGURE 4 A beam of light passing through a parallel-sided glass block. Its direction changes as it passes from the air to the glass, and then changes back again in passing from the glass to the air.

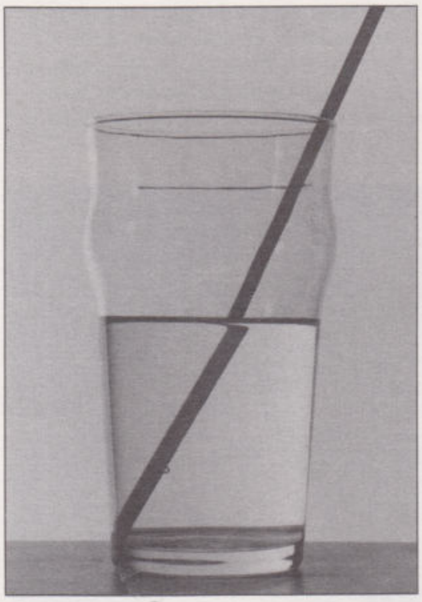


FIGURE 5 A straw placed in a glass of water appears to be bent.

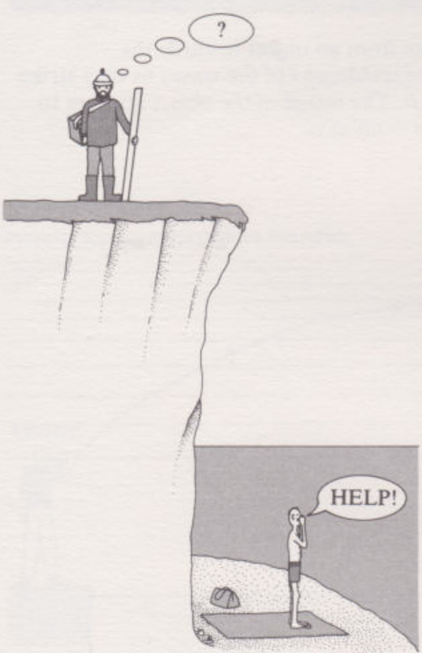


FIGURE 6 The observer on the cliff can *hear* the cry for help, even though the person who is shouting is out of sight.

above the water's surface. This causes the velocity of the sound to change continuously in such a way that the sound is redirected back down towards the ground.

Light, too, can be refracted (Figure 4). As a beam of light passes into a glass block, its direction is changed, and is then changed back again as it leaves the block of glass. It is also the refraction of light that accounts for the apparent bending of a straw placed in a glass of water (Figure 5). Again, light is behaving like a wave—it can be refracted.

So, inasmuch as light is able to transfer energy from one point to another, and can also be reflected and refracted, it would seem that a wave model of light serves us very well. However, there is another very important phenomenon associated with wave motion: the ability of waves to 'bend round corners'. The technical term for this behaviour is **diffraction**. With sound waves, for example, you know that you don't have to be in someone's direct line of sight in order to be able to hear what they are saying (Figure 6); sound can quite easily 'bend round corners'. Similarly with water waves: Figure 7 shows water waves diffracting around a breakwater; and Figure 8 shows some rather more carefully controlled examples of diffraction, this time with water ripples in a tank. In Figure 8a the waves are again shown bending around a 'breakwater' type of obstacle, whereas in Figures 8b, 8c and 8d there are rather more complicated situations. In Figure 8b the waves are travelling through a single aperture (we call this a single-slit), and in Figures 8c and 8d through double apertures (double-slits), with different spacings between the apertures.

In Figures 8c and 8d, it is much more difficult to see exactly what is happening than in 8a and 8b. The waves are certainly spreading into the space beyond the slits, but not equally in all directions. In fact, it seems that there are some directions in which the wave is very strong, and other directions in which the wave is virtually non-existent. We've attempted to indicate the directions of strong wave propagation by marking them on the diagram with arrows.

We shall sidestep, for the moment, the reason why sound and water waves diffract in this way, and ask the currently more relevant question, 'Does light behave like this?'. At first sight, it seems that the answer must be no—light appears to travel in straight lines. As you can see for yourself on a sunny day, light casts very sharp shadows (Figure 9). This observation implies that light does *not* bend round corners. However, as we'd now like

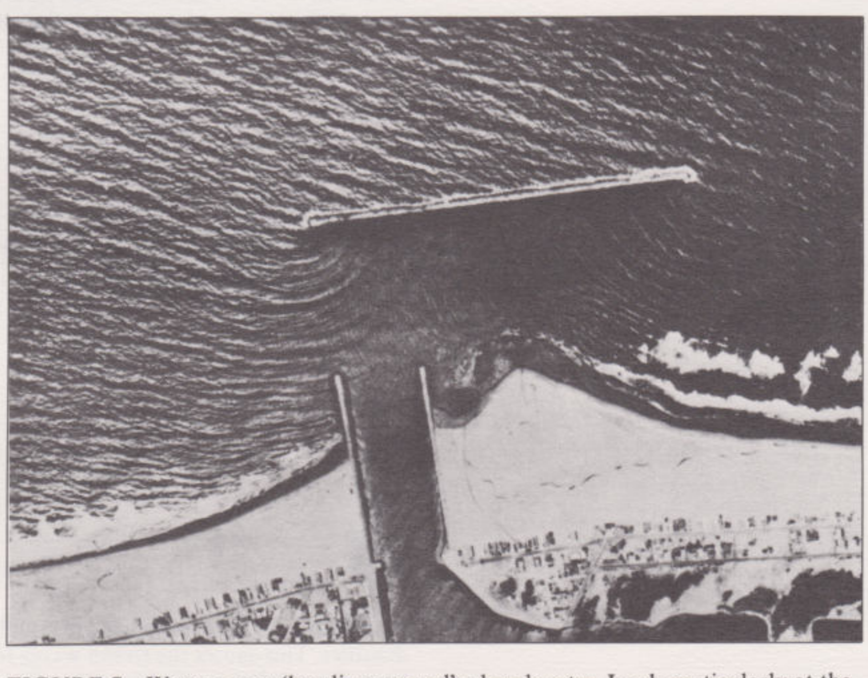


FIGURE 7 Water waves 'bending around' a breakwater. Look particularly at the waves at the edges of the obstacle.

DIFFRACTION

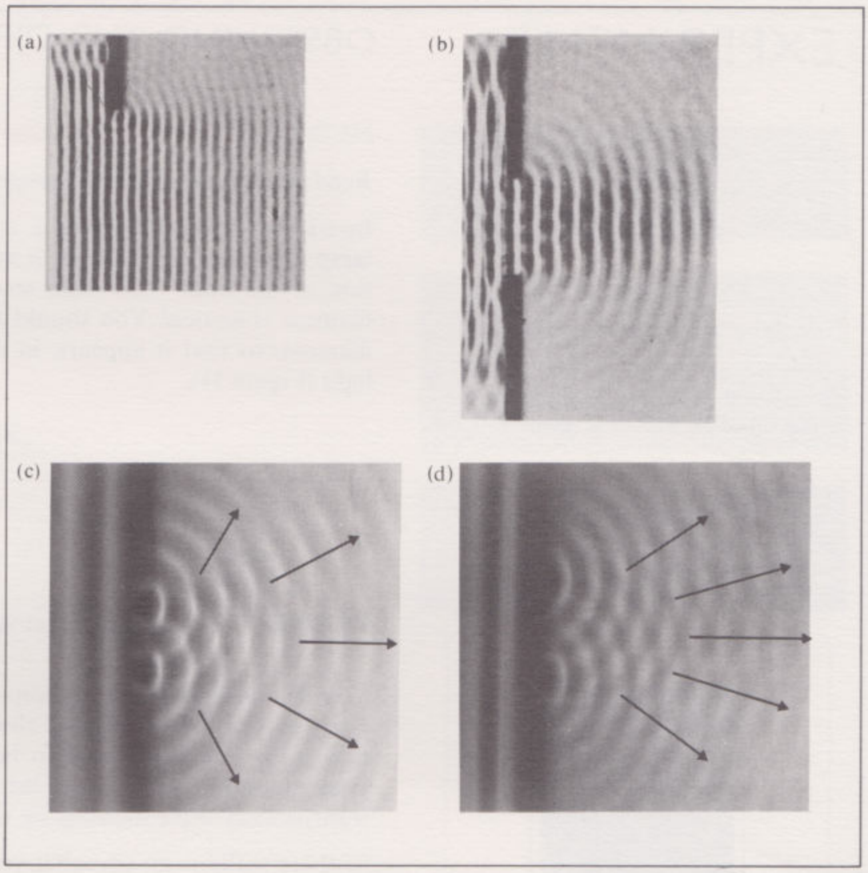


FIGURE 8 These water waves in a ripple tank must be bending around the obstacles—as indicated by the arrows—since the regions behind these obstacles are *not* in complete 'shadow'.

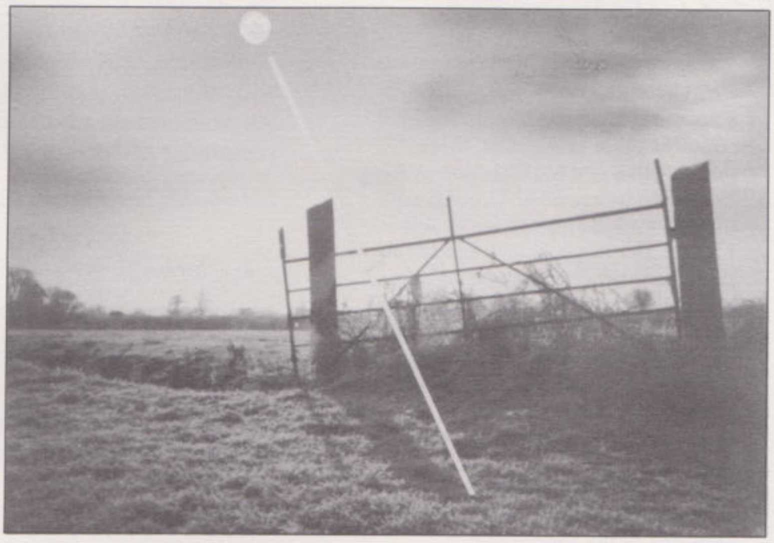


FIGURE 9 Light appears to travel in straight lines—it casts sharp shadows of objects.

you to demonstrate for yourself, it would be wrong to jump to this conclusion. Diffraction effects aren't easily observed with everyday objects because everyday objects are normally the wrong size—they're too big! If, on the other hand, we make our single- or double-slits very narrow, then diffraction effects become clearly visible.

2.2 OBSERVING THE DIFFRACTION OF LIGHT

You will be able to see the diffraction of light for yourself in the following experiment, whose results provide the basis for a large part of the remainder of the Unit. Don't be tempted to skip it!

EXPERIMENT

OBSERVING THE DIFFRACTION OF LIGHT

TIME

This experiment takes about 30 minutes

NON-KIT ITEMS

candle bulb (see Study Guide)

table lamp (or a similar light fitting that can be held so that the filament of the bulb is vertical)

KIT ITEMS

35 mm transparency (Figure 10)

green-coloured acetate filter

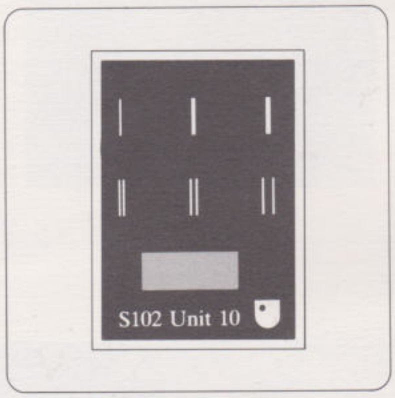


FIGURE 10 The 35 mm transparency in the Experiment Kit contains three single-slits, three double-slits, and a diffraction grating. When the letters S102 Unit 10 are upright and facing you (as in this Figure), the single-slits are in the top row, the double-slits in the middle row, and the diffraction grating at the bottom. The widths of the single-slits are, from left to right, 0.04 mm, 0.08 mm and 0.16 mm, respectively. Each slit in the double-slit pairs is 0.04 mm wide, but the *separation* of the slits in each double-slit is, from left to right, 0.08 mm, 0.16 mm, and 0.32 mm. The spacings of the slits in each double-slit have been exaggerated in this Figure.

METHOD

Read through the next two paragraphs before setting up the equipment.

Insert the candle bulb into a standard light-bulb socket on a table lamp or similar light fitting; it must be possible to adjust the orientation of the bulb. You must somehow position the bulb so that its filament is *vertical*. You should then be able to look 'edge-on' at this filament, so that it appears, in effect, to be a *vertical strip* source of light (Figure 11).

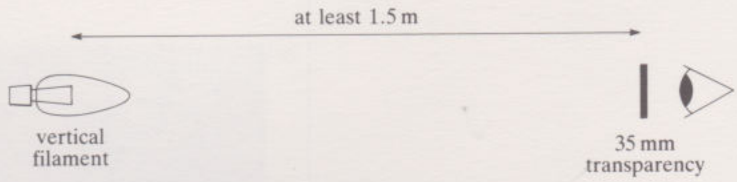


FIGURE 11 Side view of the set-up for each part of the experiment.

You will need to view this lamp from a distance of *at least* 1.5 metres. Each part of the experiment should be done *in a darkened room*. The aim is for you to be able to hold the 35 mm transparency close to your eye, and then to look through the various vertical slits on the transparency while focusing on the distant vertical strip filament.

(If there really is no way that you can set up the filament so that it is vertical, you can still perform the experiment with a horizontal filament—only now you will have to turn the transparency through 90° as well, so that the slits are horizontal. *The slits must have the same orientation as the filament.* This arrangement is nowhere near as convenient, but it will enable you to perform the experiments. You will, of course, also have to make the mental adjustment of imposing a 90° rotation on all the observed pattern orientations discussed in the remainder of Section 2.2.)

Set up this basic arrangement now before reading on.

2.2.1 SINGLE-SLIT DIFFRACTION

First read up to the end of ITQ 1 to find out what you're expected to do, and then try to do the experiment in such a way as to be able to answer the ITQ.

Locate the three single-slits on the 35 mm transparency (Figure 10). If the lettering on the transparency is the right way round for you to read it, then the widest slit is on the right, the narrowest on the left. Hold the transparency close to one eye and look through the widest slit (the rightmost slit) towards the vertical light-filament. *Focus on the filament.* You should see considerably more than just the filament. Now move the transparency so that you are looking through the middle single-slit. Note in what way the pattern you see is different from the first pattern. Finally, repeat the observation for the narrowest single-slit (on the left of the transparency). Now answer ITQ 1, noting down your observations in your Notebook. After you have completed the ITQ, compare your answer with ours before you continue.

ITQ 1 When looking through these slits you should have seen light apparently spread horizontally on either side of the direct light from the filament. The light is being 'bent round the edges' of the slit.

(a) Is the light spread out evenly in the horizontal direction, or are there some regions of darkness and some of brightness?

DOUBLE-SLIT

DIFFRACTION GRATING

(b) What colour is the light at the centre of the pattern, i.e. the direct light from the filament? What colour effects are there away from the centre of the pattern?

(c) When you move to the middle slit, are the bright and dark regions the same as they were for the wider (right-hand) slit?

(d) Does the trend continue when you move to the narrowest slit?

2.2.2 DOUBLE-SLIT DIFFRACTION

Read as far as the end of ITQ 2, before doing the experiment.

Repeat the experiment, this time looking through each pair of the **double-slit** apertures in turn. (The two slits in each double-slit pair are so close together that you cannot avoid looking through them simultaneously.) The width of each individual slit in all these double-slit apertures is the same (they all have the same width as the narrowest single-slit in the sequence above). However, the *separation* of the slit pairs is different in the three cases: 0.32 mm, 0.16 mm and 0.08 mm (Figure 10).

Begin, as in the single-slit case, with the rightmost double-slit; this is the most widely separated pair. Again, look through the vertical slits at the vertical light-filament. Then move to the middle double-slit, and then again to the leftmost double-slit. Make your observations so as to be able to answer ITQ 2. After you have completed the ITQ, compare your answer with ours before you continue.

ITQ 2 Once again, you should have observed that light apparently spreads out horizontally on either side from the filament. The following questions are essentially the same as the ones you have answered in ITQ 1.

(a) Is the light spread out smoothly on either side of the straight-through light?

(b) What colour is the light at the centre of the pattern, and how does it change, if at all, away from the centre?

(c) How does the observed pattern of light change as you move your eye first to the central double-slit, and then to the leftmost double-slit (i.e. the one with the *smallest* separation between the slits)?

2.2.3 DIFFRACTION BY A GRATING

Read as far as the end of ITQ 3, before doing the experiment.

A **diffraction grating** is a large number of equally-spaced, parallel slits, all sitting shoulder to shoulder. It's really just an alternating sequence of transparent and opaque material—a 'railing of slits', if you like.

There is a diffraction grating at the bottom of the 35 mm transparency (the slits in the grating have the same orientation as the single- and double-slits above them—they are vertical when the lettering on the transparency is upright). Hold your transparency up to your eye, and look through this grating at the vertical light-filament. Again, focus your eye on the filament, not on the grating. Make your observations so that you can answer ITQ 3.

ITQ 3 With the diffraction grating, you should once again see a horizontal spreading of the light on either side of the straight-through image.

(a) Is the pattern of darkness and light more distinct or less distinct than in the single- and double-slit cases?

(b) What colour is the straight-through image, and what colour(s) are the bright regions on each side of the straight-through direction?

EXPERIMENT CONTINUED

Finally, take the green acetate filter and hold it against the 35 mm transparency, so that as you look through the grating you are also looking through the filter at the vertical light-filament. Now go back over all the slit patterns with this filter in place, concentrating mainly on the pattern produced by the diffraction grating. In your Notebook, note the way in which the patterns are changed by the presence of the filter. *Do this experiment now before reading on.*

What you should have found, in all cases, is that the patterns of light that you observed before are still present, but now only the green colour (the colour of the filter) is present in the pattern. The spreading of the light into rainbow-like spectra has disappeared. This is particularly noticeable with the diffraction grating, where all the bright regions now appear as narrow green-coloured 'lines' and there is no overlapping of these lines towards the extremities of the pattern. Note that the straight-through bright region is also green.

The above observations can be understood once it is realized that 'white' light is actually light containing all the (rainbow) colours of the spectrum—red through orange, yellow, green and blue to violet. The diffraction effect does not split up these colours at the centre of the pattern—hence the light is still white there—but it does do so for the patterns of light away from the straight-through direction. It does this by bending red light slightly more than blue light, with intermediate bending for the colours in between, so that within each band of diffracted light (on either side of the straight-through beam) the colours are spread out. With the filter in place, the only light that gets through is green light. Hence, the straight-through light will be green, and only the green part of the spectrum will be present in each fringe. Note that it is always the case that the fringe effect—the diffraction of light caused by slits—is clearest when the light source is single-coloured. We could have used a red filter, a yellow filter or a blue filter, for example, and had the same effects with red, yellow or blue light respectively.

ITQ 4 Why did we insist in these instructions that the light-filament must have the same orientation as the slit(s)?

SUMMARY OF SECTION 2

Light does seem to have several wave properties: it can transfer energy from one place to another, it can be reflected, it can be refracted, and, as you saw in the experiment, it can be diffracted. Each of these phenomena, particularly diffraction, is characteristic of waves and wave motion. Clearly, it is well worth pursuing the wave model of light.

3 WAVE CONCEPTS

3.1 THE LANGUAGE OF WAVES (AV SEQUENCE)

CREST OF A WAVE

TROUGH OF A WAVE

WAVELENGTH, λ

AMPLITUDE OF A WAVE

PERIOD OF A WAVE

FREQUENCY OF A WAVE

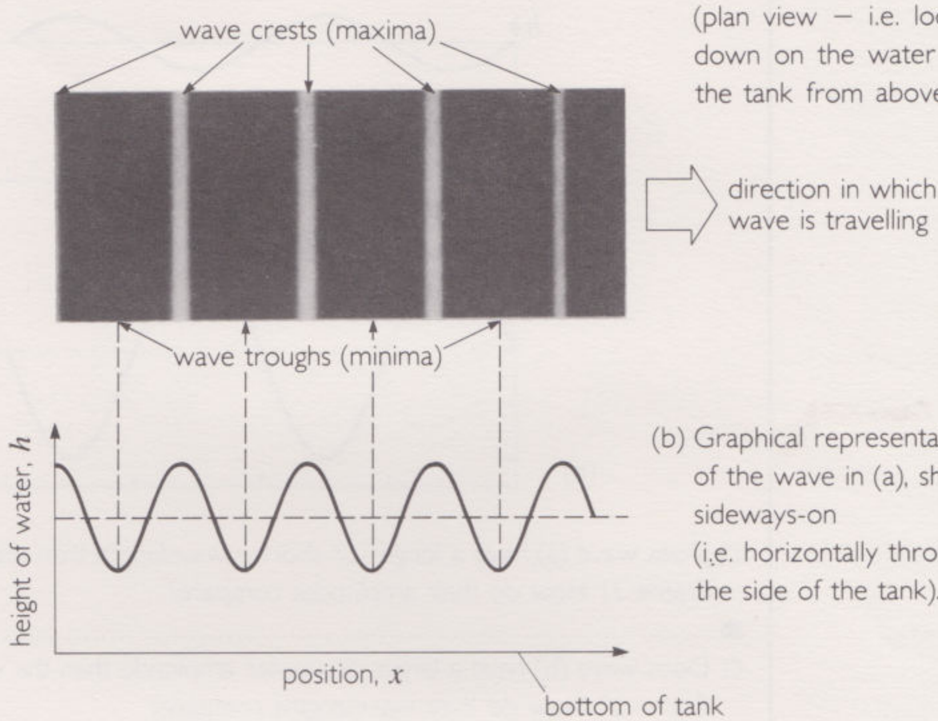
HERTZ, Hz

REPRESENTATIONS OF A WAVE

In this Section, we want to introduce you to the vocabulary and basic mathematics of waves—to show you how a wave can be characterized, and how it moves in both space and time. This will provide a firm foundation on which to build our wave model of light. However, some of the ideas in this Section may at first seem a little abstract and difficult to visualize. So, in order to make the exposition of this material as visual as possible, we have chosen to present the basic ideas in an AV sequence. You should study this sequence carefully, for it contains absolutely key material for subsequent Sections.

You should now listen to the AV sequence 'The language of waves' on Tape 2, Side 2, Band 2.

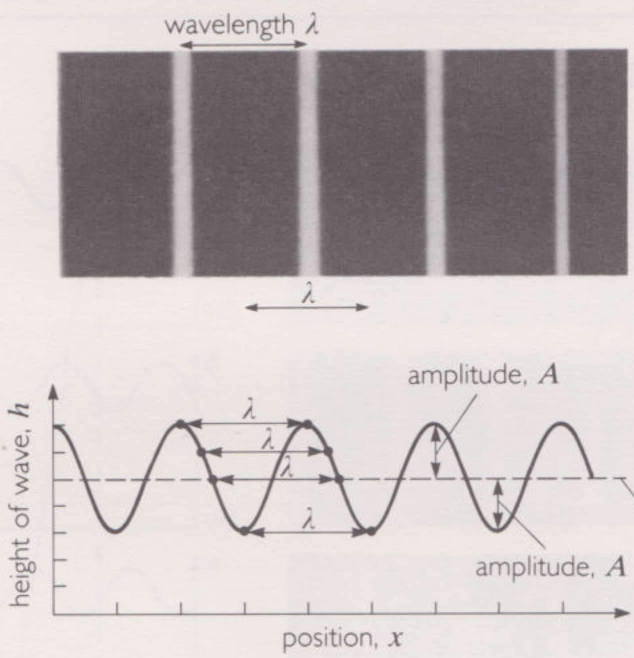
1 A water wave



(a) Ripple tank photograph (plan view – i.e. looking down on the water in the tank from above).

(b) Graphical representation of the wave in (a), shown sideways-on (i.e. horizontally through the side of the tank).

2 The wavelength and amplitude of a wave

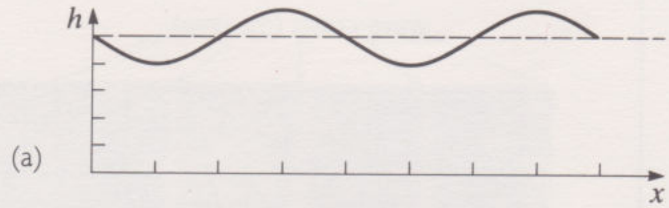


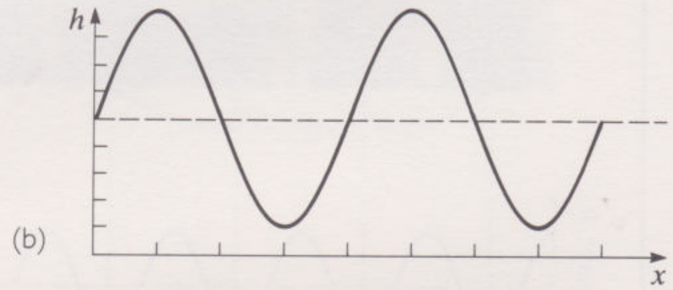
(a) The wavelength λ (i.e. the *repeat* distance of the wave).

(b) The amplitude A (i.e half the vertical distance between the crests and troughs of a wave).

$$\underbrace{\text{energy of wave}}_{\text{doesn't depend on its wavelength}} \propto (\text{amplitude})^2$$

3 Some questions





□ Does wave (a) have a longer or shorter wavelength than the wave shown in Frame 2? How do their amplitudes compare?

■ ....

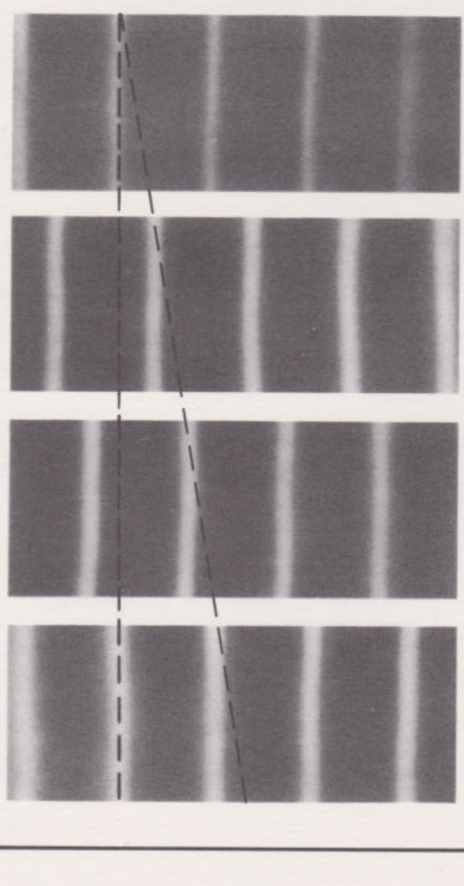
□ Does wave (b) have a larger or smaller amplitude than the wave shown in Frame 2? How do their wavelengths compare?

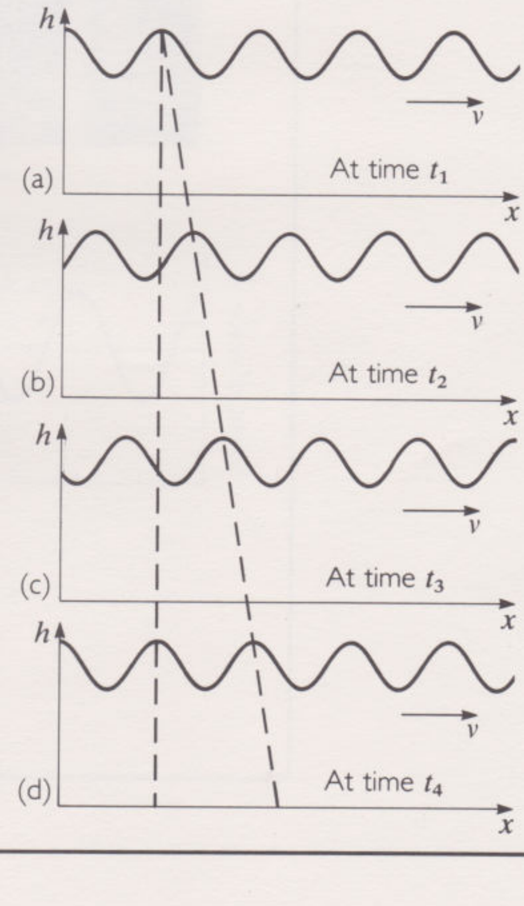
■ ....

□ How does the wavelength of wave (a) compare with that of wave (b)?

■ ....

4 The propagation of a wave





5 The period of a wave

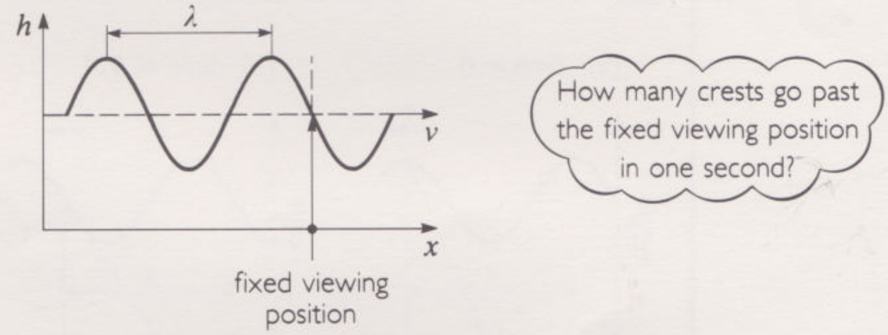
$$\text{speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

Therefore $$\text{time taken} = \frac{\text{distance travelled}}{\text{speed}}$$

Let the constant speed of a wave be v.
If T is the time for the wave to move forward by one wavelength λ, then:

period of wave — $\boxed{T = \frac{\lambda}{v}}$ — wavelength (λ); constant speed of wave (v)

6 The frequency of a wave



In T seconds, 1 cycle goes by (Frame 5).

Therefore in 1 second, $\frac{1}{T}$ cycles go by;

e.g. if $T = 4$ seconds, $1/4$ cycle goes by in one second.

The frequency of a wave is the number of waves that go by in one second.

frequency of wave — $\boxed{f = \frac{1}{T}}$ — period of wave

The unit of frequency is the hertz (abbreviated as Hz)

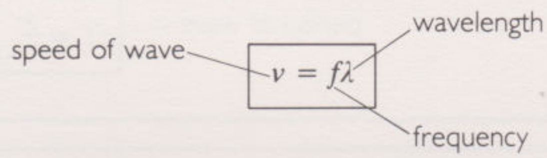
$$1\,\text{hertz} = 1\,\text{second}^{-1}$$
$$\text{i.e.}\ \ 1\,\text{Hz} = 1\,\text{s}^{-1}$$

7 The speed of a wave

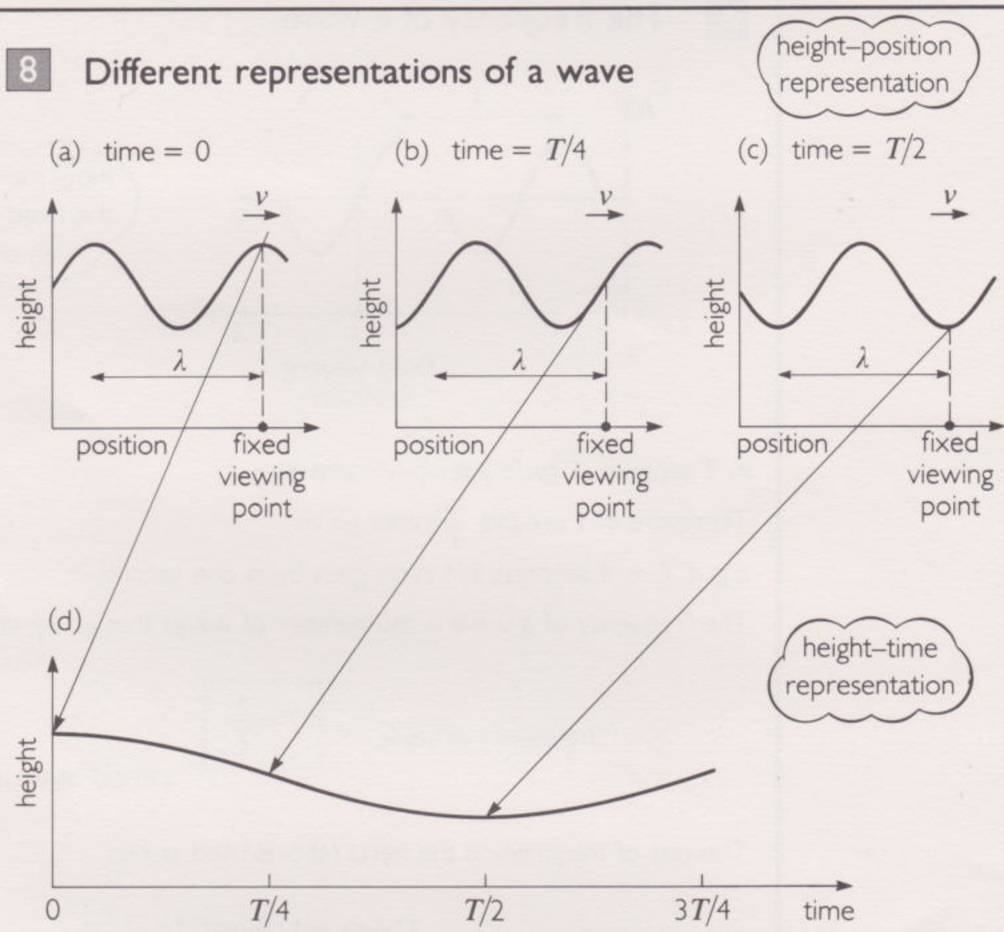
$T = \frac{\lambda}{v}$ period = $\frac{\text{wavelength}}{\text{speed}}$ (Frame 5)

$f = \frac{1}{T}$ frequency = $\frac{1}{\text{period}}$ (Frame 6)

Therefore, combining these equations, $f = \frac{1}{(\lambda/v)} = \frac{v}{\lambda}$

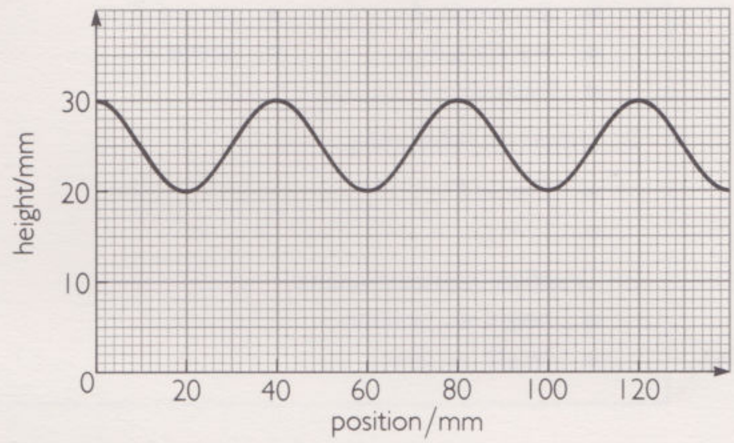


8 Different representations of a wave

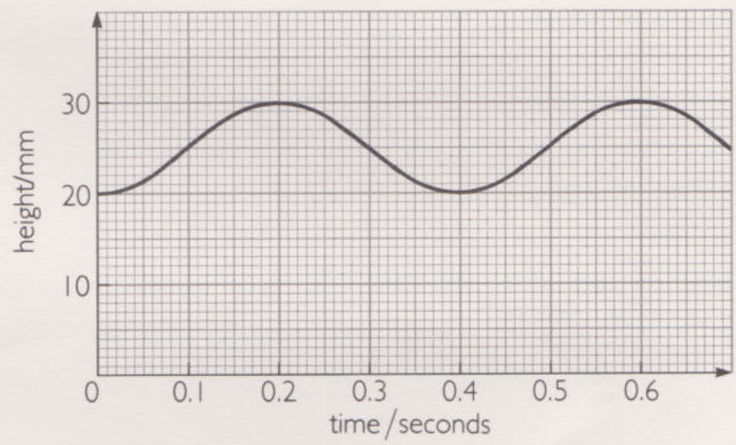


9 Some more questions

(a)



(b)



Q1 The top graph (a) in the Frame shows the height-against-*position* representation of a wave. Note that both the height and the position are given in millimetres. Can you say from this graph what
(a) the amplitude A,
(b) the wavelength λ,
(c) the frequency f and
(d) the period T
are for this wave?

Q2 The graph (b) shows the height-against-*time* representation of a wave. Can you say from this graph what
(a) the amplitude A,
(b) the period T,
(c) the wavelength λ and
(d) the speed v
are for this wave?

Q3 If graphs (a) and (b) relate to the *same* wave, is there now any more information that you can deduce about this wave?

....

....

....

10 Summary

1 Quantities needed to specify a wave:

- amplitude A
- wavelength λ
- frequency f
- period T
- speed v

2 Important relationships:

- $f = 1/T$
- $v = f\lambda$

3 Graphical representations:

- height vs. position
- height vs. time

SUPERPOSITION OF WAVES

PRINCIPLE OF SUPER-POSITION

3.2 THE SUPERPOSITION OF WAVES

3.2.1 THE PRINCIPLE OF SUPERPOSITION

Up till now, we've only been considering how a *single* wave propagates. But in most situations that we're likely to meet, there is likely to be more than just one wave—indeed, there are likely to be lots of waves 'travelling through one another' which, for a time, occupy the same space. How do these waves behave when they pass across each other? Figure 12 shows an example, albeit a rather complicated one, of exactly this situation: ripples spreading out from a number of raindrops, and passing through each other. The first thing to notice is that, after passing through a region of overlap, any set of ripples continues onwards, totally unaffected by the other sets—the ripples keep their characteristic shape. Waves do not seem to knock one another off course; they can pass through each other totally unscathed.

FIGURE 12 The ripples produced by raindrops striking a water surface are able to pass through each other without hindrance.

At first sight, this may seem to be a rather surprising property of waves. Yet without this phenomenon, the world would be a very different place: whenever you were carrying on a conversation with someone, their words would become distorted if someone else spoke across your line of communication; you would only ever see things properly when no other light wave was crossing your line of sight; you would only be able to pick up undisturbed radio and TV signals if there were no other signals crossing the path between the transmitter and your receiving aerial. Clearly this property of waves, surprising or not, is very important.

But what happens at the actual place where the waves cross—the region of overlap? The answer to this question is remarkably simple: the waves **superpose** in such a way that *the resultant disturbance at any point is simply the sum of all the contributing disturbances at the same point*. We just add the waves up point by point. This is known as the **principle of superposition**, and it is the key to understanding many wave phenomena. Let's look at how this superposition principle works in a few simple cases.

3.2.2 SUPERPOSITION OF WAVE-PULSES ON A STRETCHED ROPE

Consider, first of all, not so much a wave, as a 'wave-pulse' or more exactly, two wave-pulses travelling towards each other at the same speed along a stretched rope (Figure 13). You can imagine the rope to be something like a

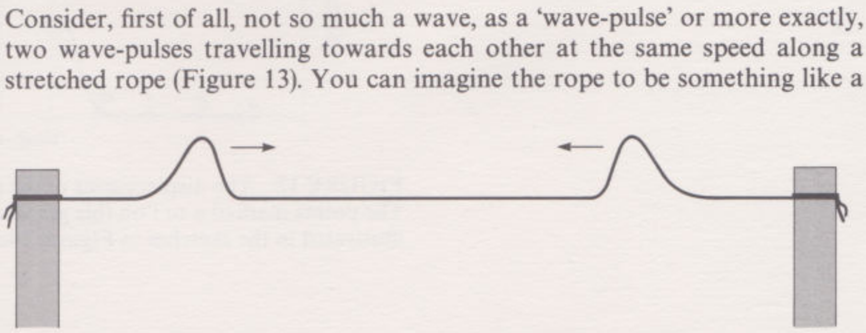


FIGURE 13 Two wave pulses travelling towards each other along a stretched rope.

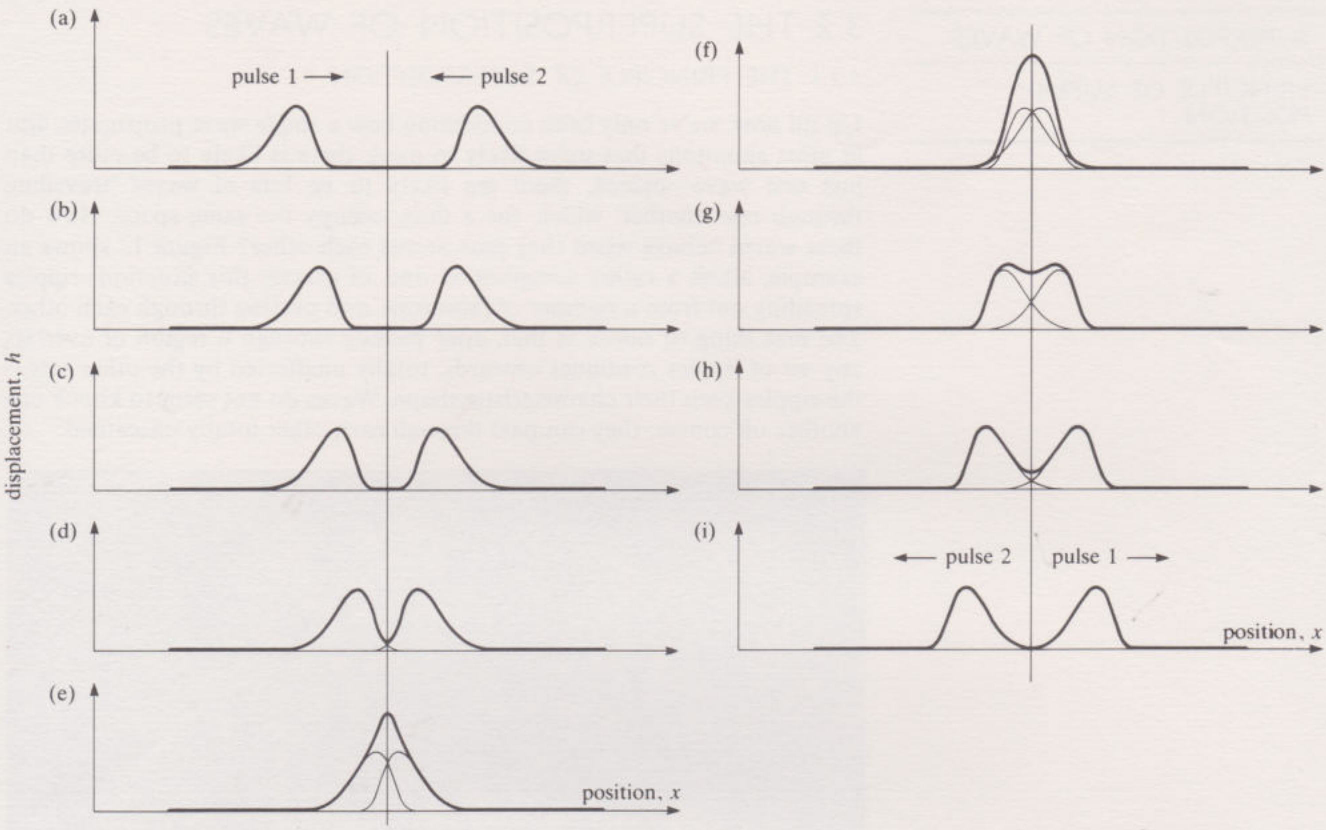


FIGURE 14 The superposition of the two wave-pulses (shown in Figure 13) as they pass through each other. The sequence of diagrams (a)–(i) are representations of the rope's state at successive, equally-spaced intervals of time. The thick line represents the actual position of the rope, while fine lines indicate the component pulses where they are being superposed. The central line indicates the midpoint of the rope.

low circus tightrope, and the pulses to have been originated by two people—one at each end of the rope—executing karate chops (upwards!) on the bit of the rope next to them. The nine sketches in Figure 14 show what happens (on displacement-against-*position* graphs) as the two pulses move towards each other with constant speed, pass through each other, and come out again at the other side. Successive graphs in this diagram (going from (a) to (i)) represent snapshots of the combined wave profile at successive, equally-spaced intervals of time.

We can now work out how the displacement at the midpoint of the rope (the point lying on the central line in each of the sketches in Figure 14) changes with *time*. Suppose the midpoint displacement in sketch (a)—it's actually zero here—represents the displacement at some arbitrary start time. Then the midpoint displacement at subsequent, equally-spaced intervals of time can easily be measured from the sequence of sketches (b)–(i) in the rest of Figure 14. The resulting displacement-against-*time* graph for the midpoint of the rope is shown in Figure 15.

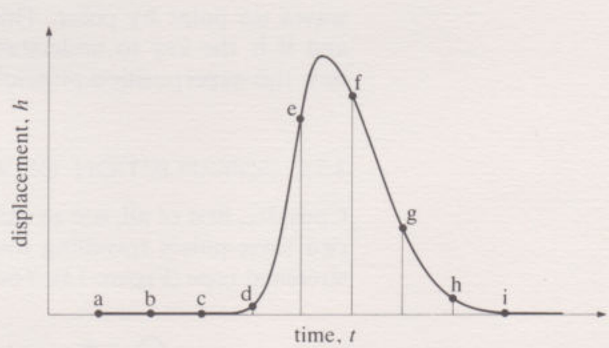


FIGURE 15 The displacement of the rope's midpoint, shown as a function of *time*. The points marked a to i on this graph correspond to the nine states of the rope illustrated in the sketches in Figures 14a to 14i, but not on the same scale.

Let us spell out the steps we've taken. We added the two waves together at *all points in space*, for a discrete number of successive time intervals. We then worked out the displacement of the midpoint as a function of time, from the *sequence* of displacement-against-position sketches.

There is an alternative way of approaching this problem. Although this approach does not have much more to offer in this particular case, it is often easier to cope with in more complicated examples. This approach begins from a different standpoint. We ask, first, how the midpoint of the rope would change with *time* if each pulse were to *travel across this point by itself*; this gives us two graphs of displacement against time (one for each pulse alone), which we then add together.

Figures 16a and 16b illustrate how this second approach works for the case of the two pulses on the rope. Don't be misled by the apparent reversal of the profile for pulse 1 in Figure 16a: the actual pulse shape on the rope itself is shown in Figures 13 and 14 as variation with respect to *position*, whereas Figure 16a shows the rope's midpoint variation with respect to *time*. The steep edge of pulse 1 will reach the midpoint before the less steep 'tail' of the pulse, so the steep increase in displacement at the midpoint will occur *earlier* (i.e. at smaller values of time) than the less steep return to zero displacement. The combined effect of the two pulses passing through the midpoint together is now determined, from the principle of superposition, by simply adding the two graphs together at all time values. This is shown in Figure 16c. The end result is the same, whichever of the two alternative approaches we adopt.

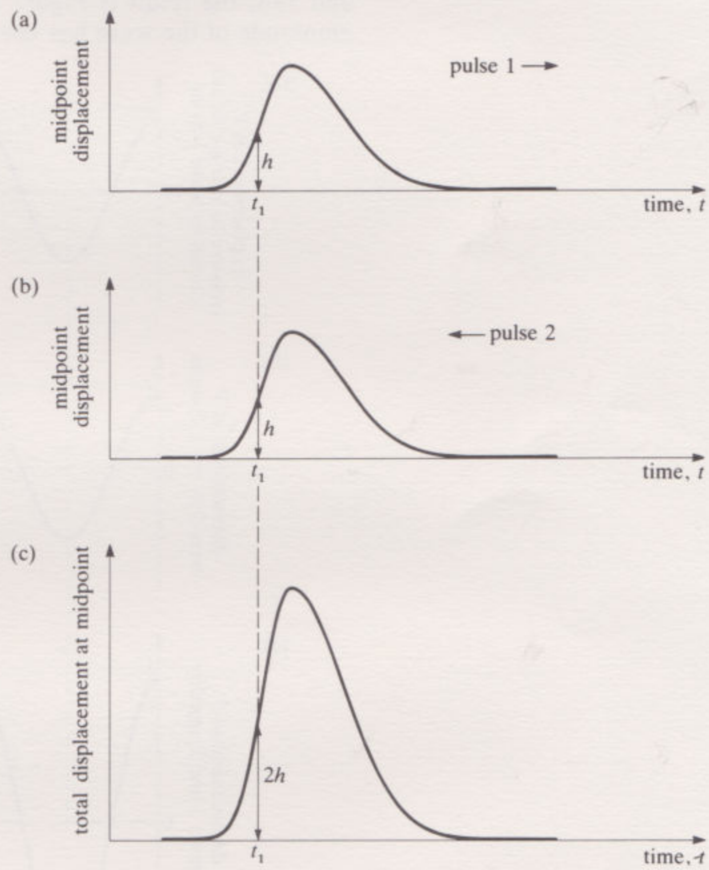


FIGURE 16 (a) How the displacement of the rope's midpoint would change with time, if pulse 1 (moving from left to right) were to traverse this point alone. (b) The same effect produced by pulse 2 (moving from right to left). (c) The combined or resultant displacement, shown at all time values. This graph is simply the *sum* of the two contributing graphs. For example, at the time t_1 (indicated on all three graphs), the combined displacement in graph (c) is $2h$; it is the addition of the two displacements h in each of the graphs (a) and (b) due to the separate pulses. This Figure is on the same scale as Figure 15.

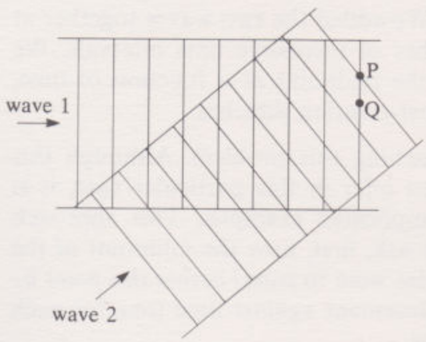


FIGURE 17 A 'snapshot' showing the plan view of two waves (travelling with the same speed) crossing each other at an angle. The lines represent the crests of the waves—the troughs must therefore be half-way between the lines. At P, a crest from wave 1 coincides with a crest from wave 2; at Q, a crest from wave 1 coincides with a trough from wave 2.

3.2.3 SUPERPOSITION OF TWO CONTINUOUS WAVES

The second of the two approaches used above is by far the more straightforward in the rather more complicated example shown in Figure 17. Here, we have two continuous waves (like the water waves in the ripple tank) combining with each other at an angle. Again, we assume that the two waves travel at the same speed. Suppose that we want to know how the displacement varies with time at the point marked P in the Figure. To keep the argument as simple as possible, we shall assume that the waves each have the same amplitude and the same wavelength.

First, ask yourself how the displacement at P would change *if only wave 1 were present*. Do you agree that the wave height would oscillate up and down, as first a crest, and then a trough, and then another crest, and so on, passed through P? This is represented by Figure 18a. If only wave 2 were present, then exactly the same argument would apply, and the displacement, shown as a variation with time, would be as in Figure 18b. Note that we have called the particular instant of time illustrated in the Figure 17 snapshot, $t = 0$. Thus, in both Figures 18a and 18b, the displacement is a maximum at $t = 0$ because in Figure 17 both waves have a crest passing through P at this instant. In other words, at $t = 0$ the two waves are *in step* with each other at P—we say that the waves are in **phase** at point P. But as you can see from Figures 18a and 18b, as the two waves pass through the point P they remain in phase *at all times* (because they travel at the same speed and have the same wavelength).

Now, to find the resultant wave at P, we simply add together Figures 18a and 18b; the result is Figure 18c, in which you can see that the effective amplitude of the wave has been doubled. This effect is generally known as

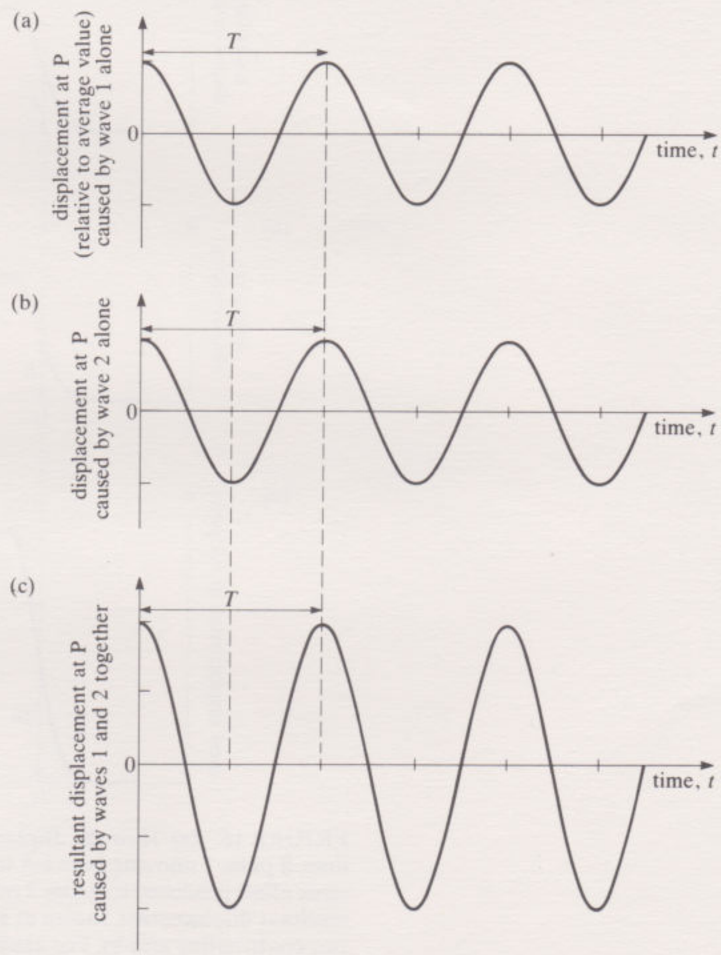


FIGURE 18 The variation of displacement at P *with time* for (a) wave 1 alone, (b) wave 2 alone, (c) waves 1 and 2 together. Waves 1 and 2 have the same amplitude, the same wavelength, and are in phase, and hence combine to give a resultant wave that has twice the amplitude of the component waves. This is called constructive superposition of the waves.

PHASE (WAVES)

CONSTRUCTIVE SUPERPOSITION

CONSTRUCTIVE INTERFERENCE

DESTRUCTIVE SUPERPOSITION

DESTRUCTIVE INTERFERENCE

constructive superposition (or **constructive interference**) of the waves—constructive, because the waves add to construct something even bigger than either of the original waves.

However, there is another side to this coin: consider what happens at point Q in Figure 17. Once again, each wave alone will give rise to the sinusoidally shaped variation of displacement with time, just as in Figure 18. But there is now one big difference. In Figure 17, at our agreed time $t = 0$, a *crest* from wave 1 coincides with a *trough* from wave 2—the waves are no longer in step. In fact, they are always exactly out of step—out of phase. This situation is illustrated in Figures 19a and 19b. Crests in (a) coincide with troughs in (b), and vice versa. And now a very remarkable thing happens when we add 19a and 19b together. For, at every value of time t, the displacement in (a) is always exactly counteracted by an equal but opposing displacement in (b). At Q, *the resultant displacement* (away from the average displacement value) *is, at all times, zero*. This phenomenon is known as **destructive superposition** (or **destructive interference**). Two waves are adding together in such a way as to cancel out each other's displacements!

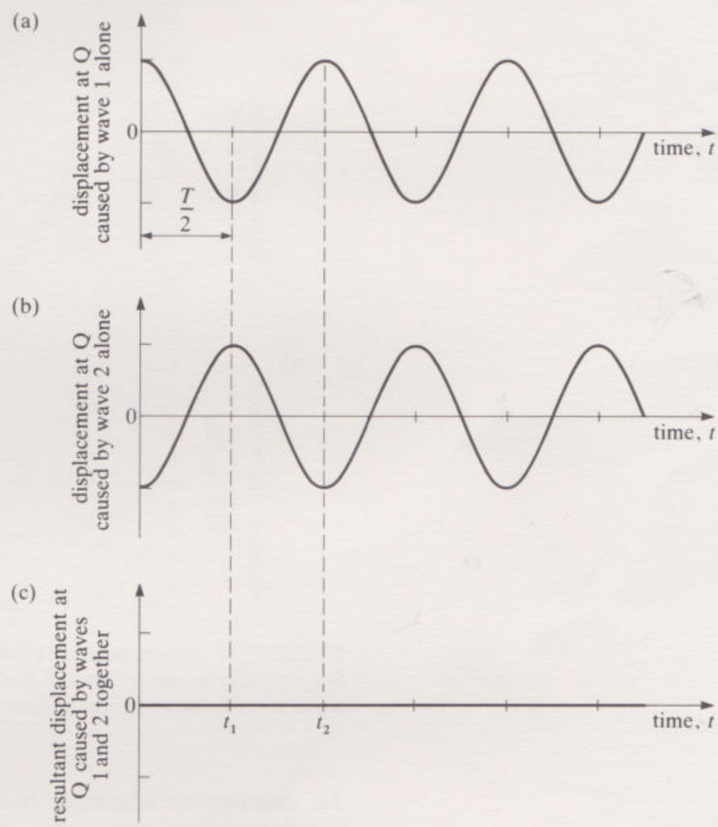


FIGURE 19 How waves 1 and 2 add together at point Q in Figure 17. The two waves have the same wavelength and amplitude, but are completely out of phase, and hence combine to give zero displacement (from the average displacement value) at all times. For example, at time t_1 the trough from wave 1 exactly cancels the crest from wave 2, and at time t_2 the crest from wave 1 exactly cancels the trough from wave 2. This effect is known as destructive superposition.

The situations represented by points such as P and Q in Figure 17 correspond, of course, to the two extremes—at P the waves are exactly in step, at Q they are exactly out of step. There are points in Figure 17 where some intermediate condition applies—where, say, a crest of wave 1 coincides with a zero of wave 2, or with a half-maximum value of wave 2, or with a half-minimum value of wave 2, etc. At these points, superposition will lead to *partial* cancellation or addition. There will be points in Figure 17 corresponding to all possible intermediate positions between destructive superposition and constructive superposition. ITQ 5 should help you to convince yourself that this is true.

ITQ 5 At some point R in Figure 17, a *crest* of wave 1 coincides with a *zero* displacement contribution from wave 2. Remember that waves 1 and 2 both have the same amplitude (A, say). Figures 20a and 20b show sketches of the way in which the displacements due to waves 1 and 2, respectively, change with time at this point. On Figure 20c, give a sketch of what the resultant displacement will look like at this point. (Remember, the resultant wave is given by the sum of waves 1 and 2.) Is the amplitude of the resultant wave equal to, greater than or less than the amplitude of waves 1 and 2? How does this resultant amplitude compare with that at point P?

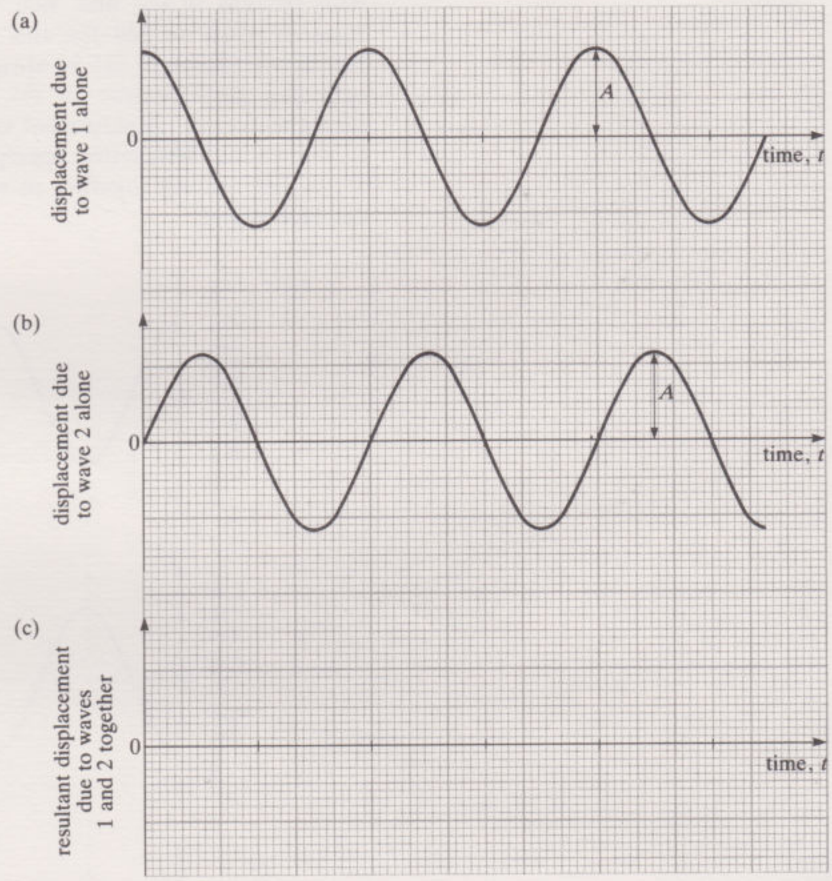


FIGURE 20 At some point on Figure 17, wave 1 is at a maximum when wave 2 *has zero displacement*. Wave 1 is shown in (a). Complete the diagram by sketching wave 2 in (b), and adding the two waves together in (c).

To summarize: whenever two waves, with equal amplitude A and equal wavelength, are superposed, the resultant combined wave (which will have the same wavelength as the component waves) can have *any* amplitude between zero and $2A$, depending on the phase relationship—the degree of 'in-stepness'—of the two waves. At the extremes, the resultant amplitude will be $2A$ when the waves are exactly in phase, and zero when they are out of step by exactly one-half of a cycle (that is, by half a wavelength).

3.2.4 SUPERPOSING WAVES OF DIFFERENT AMPLITUDE

Up till now, we've only considered the superposition of waves with the *same* wavelength and amplitude. But the principle of superposition has quite general validity: it also applies to waves that don't have the same wavelength and amplitude.

As an example of the generality of this principle, we shall consider in a bit more detail what happens when two waves with *different amplitudes* (but the same wavelength) are added together. This case is illustrated in Figure 21, again for the two extremes of the waves being exactly in phase, and exactly

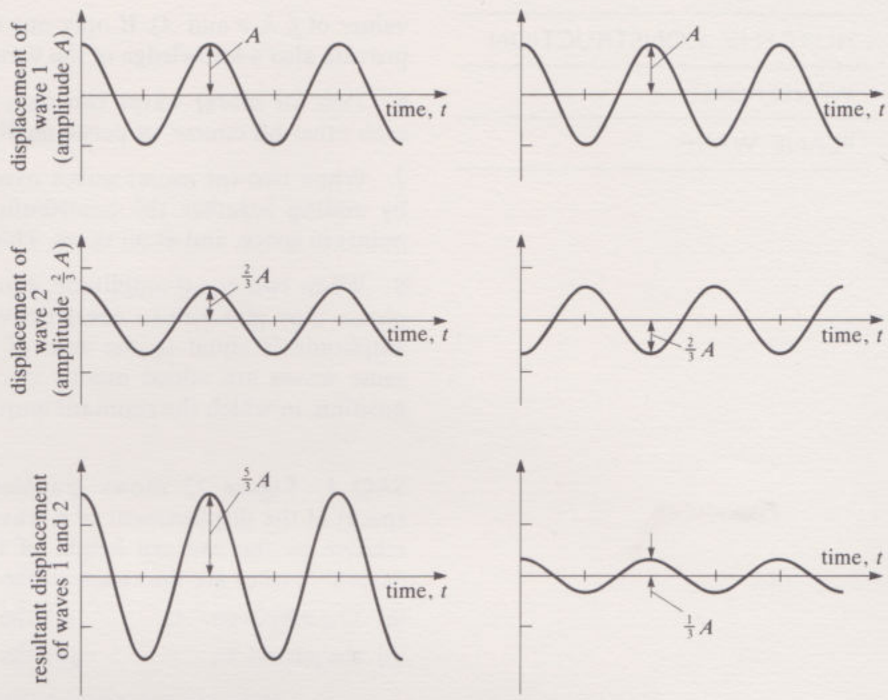


FIGURE 21 The superposition of a wave with amplitude A and a wave with the same wavelength and with amplitude $2A/3$, (a) when the component waves are in phase, (b) when the component waves are exactly out of phase. The maximum resultant amplitude obtainable corresponds to case (a) and is given by $A + 2A/3$, i.e. $5A/3$. The minimum resultant amplitude obtainable corresponds to case (b) and is given by $A - 2A/3$, i.e. $A/3$.

out of phase. As you can see from this Figure, the maximum resultant amplitude—obtainable when the waves are in phase—is the *sum* of the two component amplitudes, namely $5A/3$. The minimum resultant amplitude—obtainable when the waves are exactly out of phase—is the *difference* of the two component amplitudes, namely $A/3$. What distinguishes this example from the one in which the waves had equal amplitudes, is that here it is *not* possible to have complete cancellation—destructive superposition. In all other respects, the situations are equivalent.

SUMMARY OF SECTION 3

1 The wavelength λ of a wave is the repeat distance, measured from any point on a wave cycle to the equivalent point on the neighbouring wave cycle.

2 The amplitude A of a wave is the distance from the average height or displacement level, to the maximum (or minimum) height or displacement level. Consequently, it is half the maximum-to-minimum height value.

3 The period T of a wave is the time interval that elapses before the wave cycle begins to repeat itself.

4 The frequency f of a wave is the measure of how many wave cycles pass a given point in a second. The unit of frequency is the hertz, Hz (equivalent to cycles per second, or simply seconds^{-1}). The frequency is the reciprocal of the period (i.e. $f = 1/T$), and it is related to the wavelength by the equation $v = f\lambda$, where v is the speed of the wave.

5 It is possible to represent waves graphically in two different ways. Either the variation of displacement with *position* can be shown at a single instant of time, or the variation of displacement with *time* can be plotted at a single point in space. These two representations are complementary: both are needed if complete information about the wave is to be provided (i.e. the

HUYGENS' CONSTRUCTION

WAVEFRONT

PLANE WAVE

values of f, λ, v and A). If only one representation is used, it is necessary to provide also a knowledge of the wave's speed v.

6 Two (or more) waves can pass through each other without 'knocking each other off course' or permanently distorting each other.

7 When two (or more) waves overlap, the resultant disturbance is found by adding together the contributions from the component waves at all points in space, and at all times. This is the principle of superposition.

8 When two equal-amplitude, equal-wavelength waves are superposed in phase, they give rise to constructive superposition, in which the resultant amplitude is equal to the sum of the component amplitudes. When the same waves are added exactly out of phase, they give destructive superposition, in which the resultant amplitude is zero.

SAQ 1 Figure 22 shows graphically the variation (at a fixed point in space) of the displacement of a wave with time. The displacement is shown relative to the average height of the wave. If the speed of this wave is $5.0\,\mathrm{m\,s^{-1}}$, what are the values of the following features of the wave?

(a) The amplitude A; (c) the frequency f.

(b) the period T; (d) the wavelength λ.

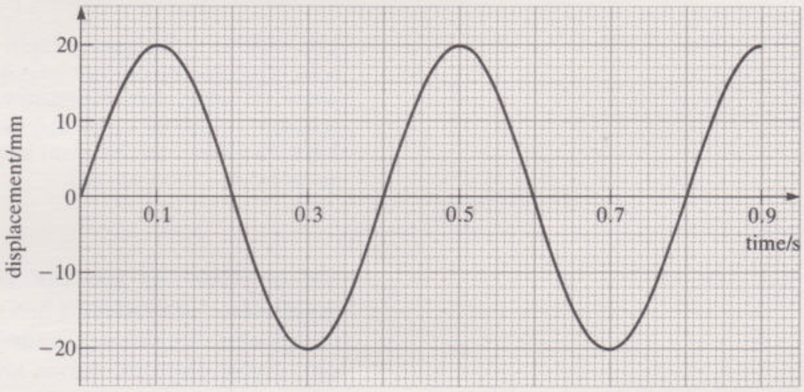


FIGURE 22 The variation of a wave's displacement with time. The wave's speed is $5.0\,\mathrm{m\,s^{-1}}$ (see SAQ 1).

SAQ 2 A storm at sea produces two different kinds of deep-water wave: one with wavelength 17 m and frequency 0.30 Hz, the other with wavelength 2.0 m and frequency 0.90 Hz. Calculate the speeds of the two kinds of wave, and comment on what would be seen on a shoreline 100 km from the storm.

SAQ 3 Figure 23 shows the displacement of two waves of the same frequency but different amplitude, at a point where the waves are out of phase. Sketch on the Figure the resultant displacement that is generated when the two waves are superposed. (*Hint*: Pick points on the graphs that are easy to add.)

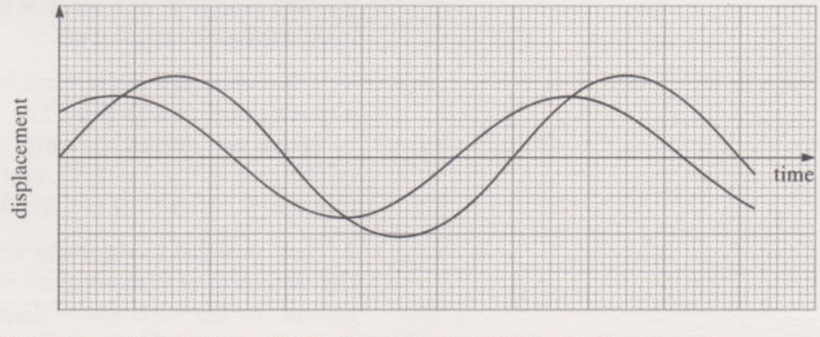


FIGURE 23 What is the resultant displacement when these two waves are superposed? (For use with SAQ 3.)

4 DIFFRACTION AND THE WAVE MODEL

Our previous discussions of the language and characteristics of wave motion, and of the ways in which waves combine with one another to produce a resultant wave, have all been directed principally to one main end—to provide the concepts necessary for understanding the phenomenon of diffraction in terms of a wave model. But we don't just want to understand *why* waves diffract; we would also like to explain the patterns of brightness and darkness that you saw in the experiment, when light was diffracted by a single-slit, a double-slit and a grating (i.e. a 'multiple-slit'). Our explanation should, of course, apply to other types of wave, but we are particularly interested, in this Unit, in its application to light waves.

4.1 HUYGENS' CONSTRUCTION

One of the first suggestions as to how diffraction could be understood in terms of wave properties, was put forward in 1678 by the Dutch scientist Christiaan Huygens. His suggestion is nowadays known by the name **Huygens' construction** (or Huygens' principle). In order to explain how Huygens' construction works, we must first introduce the concept of the wavefront.

A **wavefront** of a wave is a surface over which the phase of the wave is constant. This idea can easily be applied to a water wave in a ripple tank (Figure 24a), the type of wave you met earlier in the AV sequence. The associated wavefronts of such a wave are simply parallel planes (flat surfaces) that are perpendicular to the wave's direction of motion and that move with a speed that is the same as that of the wave. It is hardly surprising that a wave like this is usually called a **plane wave**.

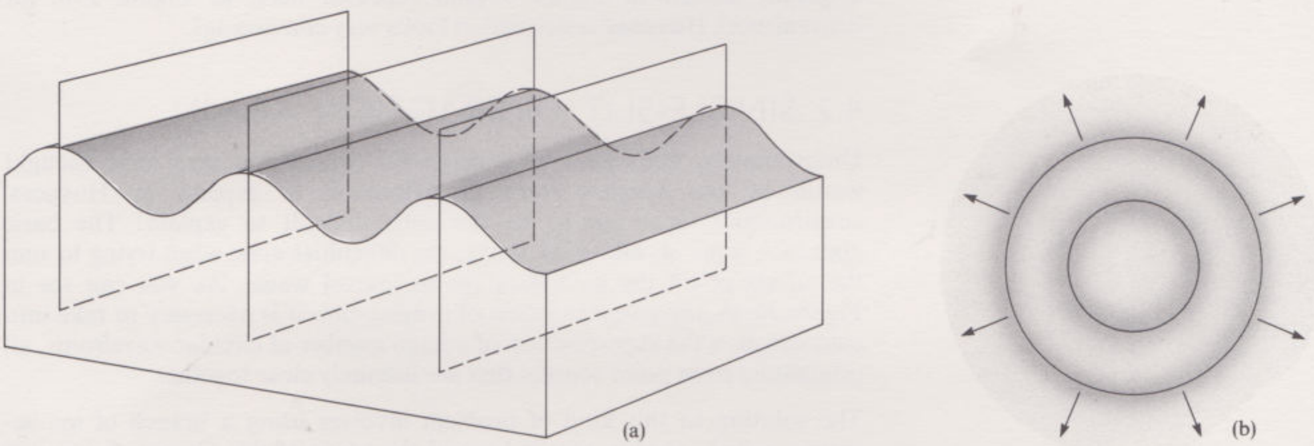


FIGURE 24 Wavefronts of (a) a plane wave, and (b) a circular wave.

Now consider a different type of wave, the type generated when for example a stone is dropped on the calm surface of a pond (Figure 24b). The wavefronts of this type of wave are circular. Keeping this situation in mind, it is fairly easy to visualize a similar type of wave, a spherical wave, whose wavefronts are spherical and which each have a radius that gradually increases with time.

Let's now use these ideas to explain how Huygens' construction works. He asserted that *every point on a wave's wavefront can be thought of as a point source of secondary circular* (*or, in three dimensions, spherical*) *waves that have the same speed and the same wavelength as the original wave.*

To find the next wavefront in the direction of propagation, you have to use the principle of superposition to add up all the contributions (in the forward direction) from the point sources on the original wavefront. This sounds as though it is a very complicated operation, and very often it is. But it's not very difficult to get the general idea of what is going on.

Consider once again the breakwater type of barrier, and look at Figure 25a. Huygens' construction has been used to 'replace' the plane wavefront that has just reached the barrier, with a series of point sources of waves. These point sources are shown in the Figure as a series of dots along the original wavefront. (In principle, there should be an infinite number of points, but for obvious reasons we have only shown a few of them here.) The circular arcs drawn around each of these points then represent the wavefronts of the *secondary* waves as they propagate in the forward direction. Now, at points such as P, well away from the end of the barrier, all these secondary waves

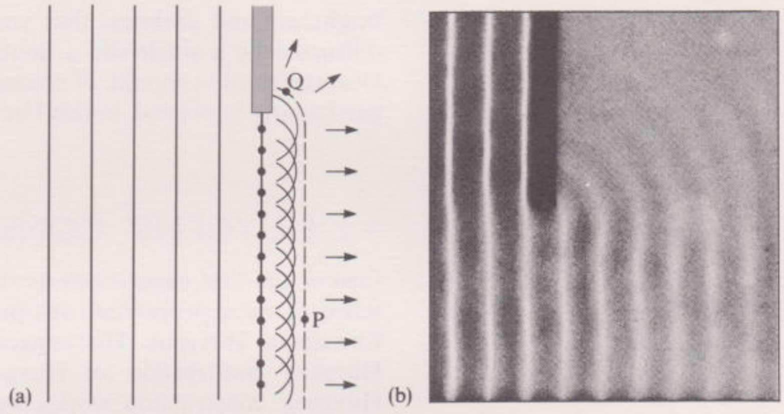


FIGURE 25 (a) Huygens' construction, used to explain diffraction by a breakwater type of barrier. Each point on the wavefront gives rise to circular secondary waves. When these are all superposed, they give rise to the dashed wavefront shown.
(b) The ripple-tank photograph of diffraction round a barrier provides strong support for Huygens' construction.

will superpose to produce another *plane* wavefront once again. But at a position near the barrier—a position such as Q— the superposition will be unbalanced, for there are no point sources in the barrier. The consequence of this is that the original wave can now extend into the 'shadow' region behind the breakwater. The dashed line shows the position of the new wavefront, as predicted by Huygens' construction. Obviously, the argument can be repeated to predict the position of the next wavefront, and then the next, and so on. When you compare this with the ripple tank photograph originally shown in Figure 8 (and repeated here, as Figure 25b, for convenience), Huygens' construction looks very convincing!

4.2 SINGLE-SLIT DIFFRACTION AGAIN

Unfortunately, diffraction by a single-slit (which you may have thought would be the simplest form of diffraction to explain by Huygens' construction) turns out to be the most difficult to explain! The basic approach is just as discussed above; the difficulties arise when trying to sum the effects of all the secondary (point-source) waves. As you can see in Figure 26, in any given direction of propagation it is necessary to take into consideration the superposition of a huge number of circular wavefronts, all originating from point sources that are infinitely close together.

The solution to this kind of problem involves using a branch of mathematics, called calculus, that is beyond the scope of this Course. So, in just

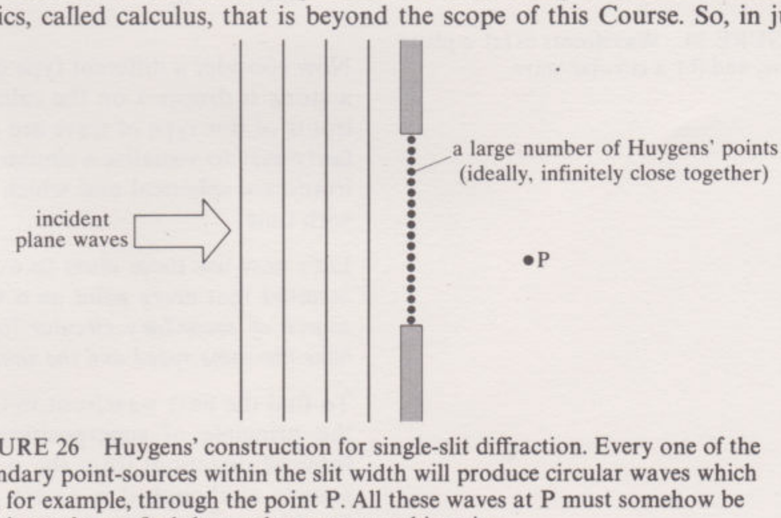


FIGURE 26 Huygens' construction for single-slit diffraction. Every one of the secondary point-sources within the slit width will produce circular waves which pass, for example, through the point P. All these waves at P must somehow be added together to find the resultant wave at this point.

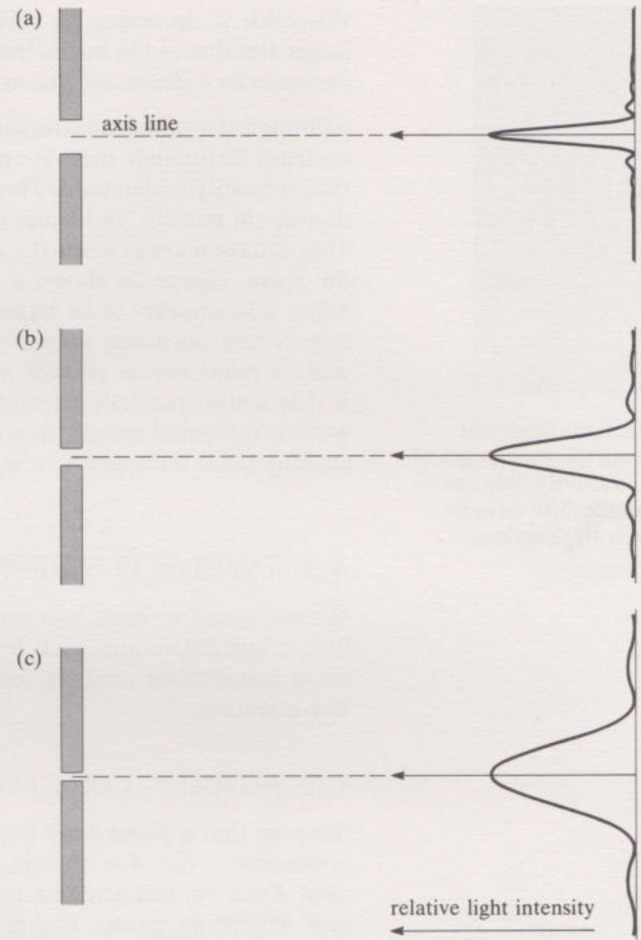


FIGURE 27 The single-slit diffraction pattern, observed a long way away from the slit, comprises a central 'band' of light flanked on each side by a sequence of regions of darkness and light. The flanking regions of brightness each have half the width of the central bright region. (a) When the slit is broad, the central bright region is relatively narrow and the flanking bright regions are close together. (b) As the slit width is reduced, the central bright region in the diffraction pattern broadens, and the flanking bright regions are further apart. (c) For narrow slits, the central region of the diffraction pattern is very broad, and the flanking bright regions are well spaced.

this one instance, we ask you to take our word for the results that are obtained. They are summarized in Figure 27. The sketch to the right of each slit is meant to indicate how the relative light intensity varies in the direction perpendicular to the axis of the slit, i.e. how much light you would see as you shifted your angle of viewing sideways when looking through the slit. As you can see, the narrower the width of the slit, the broader the central region of brightness. Furthermore, whatever the width w of the slit, the width of the central region of brightness in the pattern is always *twice* that of the flanking areas of brightness (Figure 27). This should be in accordance with the observations you made for yourself earlier. (Try the single-slit experiment again if you're not convinced.)

It's worth noting the expression predicted from the Huygens' construction for the angular extent of the *central* diffraction maximum. The angle ϕ_1 which specifies this extent (Figure 28) turns out to be given by

$$\sin \phi_1 = \lambda/w \tag{1}$$

where λ is the wavelength of the incident light and w is the width of the slit. For light with a given wavelength, notice that Equation 1 shows that as the

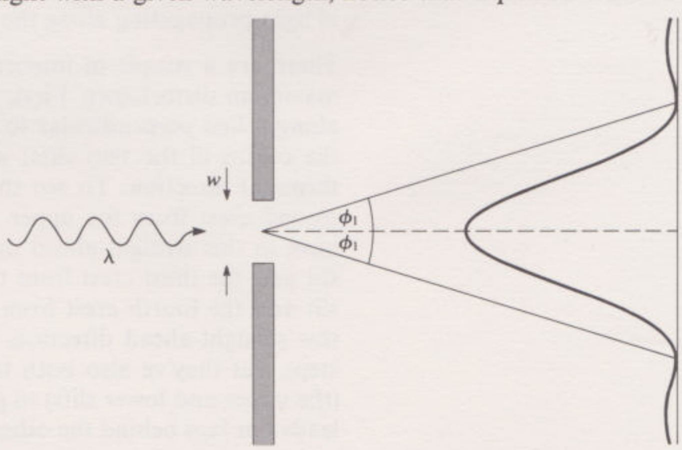


FIGURE 28 The angle ϕ_1 of the diffraction minimum is given by $\sin \phi_1 = \lambda/w$, where λ is the wavelength of the incident light and w is the width of the slit. This result can be proved using Huygens' construction and the mathematical technique of calculus.

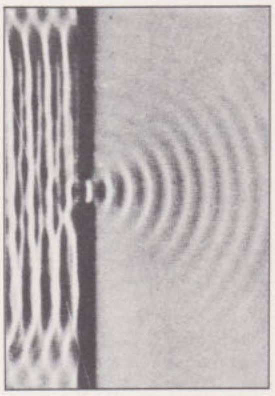


FIGURE 29 When the single-slit width is smaller than the wavelength of the light, there is effectively only one Huygens' point source. The wave is diffracted equally in all directions beyond the slit.

slit-width w decreases, so $\sin\phi_1$ increases. But for angles up to 90°, the larger the sine of the angle, the larger the angle. It therefore follows that ϕ_1 *increases* as w *decreases*, just as Figure 27 indicates.

Although it's rather unsatisfactory to have to ask you to take these results on trust, fortunately there is one particular situation in single-slit diffraction that *is* readily understood. This in turn can be made use of in explaining the double-slit pattern, and hence the pattern produced by a diffraction grating. This situation arises when the width of the slit is less than the wavelength of the wave. Figure 29 shows a ripple tank photograph of just such a case. What now appears to be happening is that the slit is so narrow that, to all intents and purposes, we can consider there to be only *one* Huygens' secondary point source present within the slit-width. The consequence of this is that almost perfectly circular wavefronts emerge from the slit; that is, the wave is diffracted equally in all directions to the right of the slit. This is our starting point for understanding two-slit diffraction.

4.3 TWO-SLIT DIFFRACTION AGAIN

We are going to have two attempts at explaining two-slit diffraction: the first, a qualitative approach based on Huygens' construction; the second, a more quantitative analysis, leading eventually to a very important diffraction equation.

4.3.1 HUYGENS' CONSTRUCTION AND THE DOUBLE-SLIT

Suppose that a plane wave impinges on two very narrow slits, separated by a distance d (i.e. d is the shortest distance between the midpoints of the slits). Each slit will give rise to identical sets of circular waves, and the two sets will be in phase. Figure 30 shows an instantaneous snapshot of a sequence of the crests of the two waves, numbered 1, 2, 3, etc.

According to the principle of superposition, the resulting wave disturbance beyond the two slits is the sum of the two emergent waves. Look at point P on Figure 30, for example. Here, the third crest of the circular wave from the upper slit coincides with the fourth crest of the wave from the lower slit. The amplitude of the local disturbance at P is the sum of the amplitudes of the two waves. At the same instant, there will be many other points at which a crest of one wave coincides with a crest of the other wave; these are indicated by the solid circles in Figure 30. As you can see, the points at which the crests of the two waves add (constructive superposition) lie on lines that 'radiate' outwards from the centre of the double-slit obstacle.

Remember that Figure 30 is a snapshot, taken at a fixed instant of time. A similar snapshot, taken at some other time, may show the first circular wavefronts a bit closer to or further away from the slits; but since the two sets of waves are of the same wavelength and emerge from the slits in phase, the pattern of superposition remains the same—the crests will always cross (i.e. constructive superposition will occur) along the same arrowed lines. If the waves in this example are light waves, then there would be bright beams of light propagating along the directions of these lines.

There are a couple of important points to note about these directions of maximum disturbance. First, there will always be one such direction lying along a line perpendicular to the plane of the barrier and passing through the centre of the two slits; we'll call this the straight-ahead (or straight-through) direction. To see this, look again at Figure 30. Notice that the second crest from the upper slit and the second crest from the lower slit meet in this straight-ahead direction; so do the third crest from the upper slit and the third crest from the lower slit, the fourth crest from the upper slit and the fourth crest from the lower slit, and so on. In other words, in this straight-ahead direction, not only are the two waves in phase (i.e. in step), but they've also both travelled the same distance from their origins (the upper and lower slits) to get to any point on this line. One wave neither leads nor lags behind the other.

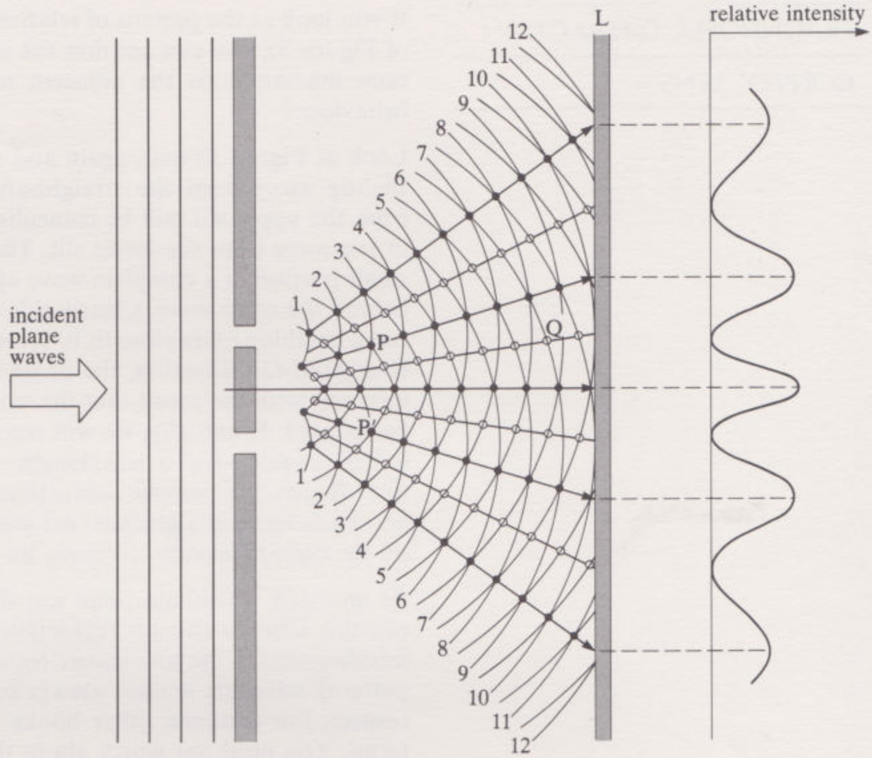


FIGURE 30 A plan view of two-slit diffraction. The circular waves from each slit superpose. In some directions (marked by the arrows and rows of solid circles on the diagram) the superposition will be constructive, giving rise to strong wave propagation. Midway between these directions (indicated by rows of open circles), the superposition will be destructive, giving rise to no wave propagation. Constructive superposition occurs when the path difference from the two slits is a whole number of wavelengths (i.e. λ, 2λ, 3λ, 4λ, etc.); destructive superposition occurs when the path difference is an odd number of half wavelengths (i.e. $\lambda/2$, $3\lambda/2$, $5\lambda/2$, $7\lambda/2$, etc.). The graph on the right shows the relative intensity of the diffraction pattern observed along the line L.

On the other hand, in the first directions of maximum disturbance on either side of the straight-ahead direction (P lies on the upper of these lines), there is always one wavelength difference between the wave from the upper slit and that from the lower slit. For example, as we have already noted, at P the third crest from the upper slit coincides with the fourth crest from the lower slit. For the exactly symmetrical point P′ on the other side of the straight-through direction, it's the other way round: the fourth crest from the upper slit coincides with the third crest from the lower slit. For all the points on each of these two lines, there is always a difference of *one* wavelength between the waves. Study Figure 30 until you are convinced.

For the next adjacent directions of maximum disturbance, crest 1 coincides with crest 3, crest 2 with crest 4, and so on. For these lines, there is always a difference of exactly *two* wavelengths between the waves. Similarly, there will be further directions of constructive superposition, not drawn in on Figure 30, where the path differences between the waves are three wavelengths, four wavelengths, and so on; these directions lie at successively increasing angles to the straight-ahead direction.

Now look at point Q in Figure 30. This lies on a *crest* of a wave from the lower slit, but *half-way between crests* of the wave from the upper slit.

☐ What happens when two waves are superposed so that the crests of one coincide with the troughs of the other?

■ The two waves superpose *destructively*. If the component amplitudes are the same, the resultant amplitude will be zero (Figure 19).

There is a whole array of points like Q lying along the same radial line from the double-slit. Along this direction we should expect destructive superposition. Virtually no energy will propagate in this direction; if the wave were a light wave, we would expect there to be darkness in this direction. Notice that because, in this direction, a crest of one wave coincides with a trough of the other wave, the two waves must be exactly out of step.

There will also be a direction of destructive superposition midway between each pair of adjacent lines of maximum disturbance. So, if we viewed a double-slit diffraction pattern from a large distance, we would expect to see a central region of brightness flanked symmetrically by nearly equally-spaced areas of darkness and light. Compare this with what you observed when you performed the experiment for yourself. Also compare Figure 30 with the double-slit ripple tank photographs in Figure 8.

FRAUNHOFER DIFFRACTION

CONVEX LENS

If you look at the pattern of relative intensity shown on the right-hand side of Figure 30, you can see that the relative intensity changes *gradually* from each maximum to the adjacent minima. How can we understand this behaviour?

Look at Figure 30 once again and consider a direction of propagation just slightly away from the straight-ahead direction. In this direction, crests from the upper slit will be coinciding with something just less than a crest in the wave from the lower slit. There will be a phase difference of only a small portion of a complete wave cycle—one wave will be lagging only just behind the other wave. Clearly this will not give constructive superposition, but something quite close to it. However, as we move further away from the straight-ahead direction, the phase differences between the two waves will increase, with the result that the 'constructiveness' of the superposition will be reduced. Eventually, we will reach that angle where the phase difference is half a cycle—half a wavelength—and the superposition will be destructive. Hence, for two-slit diffraction, the transition from darkness to light and back again is a gradual and continuous process, just as we have shown on the right-hand side of Figure 30.

As an aside, we should point out that although we have referred throughout this Unit to two-slit *diffraction* (reserving the terms superposition and interference for the *mechanism* by which waves give rise to the diffraction pattern), scientists are not always so careful in their use of language in this respect. For instance, other books often refer to 'two-slit interference patterns'. You need not worry about this, as long as you understand the concepts involved.

4.3.2 THE DIFFRACTION PATTERN AT INFINITY: THE ROLE OF THE CONVEX LENS

At various points throughout the previous discussion, we introduced the caveat 'when viewed from a large distance' (or words to that effect). This was no mere whim; it is only after travelling a large distance from the diffracting object that the waves will superpose to produce a pattern that is easy to analyse mathematically. We even give a special name to this case of 'diffraction at infinity': we call it **Fraunhofer diffraction** (after the 18th century German scientist and instrument maker, Joseph Fraunhofer).

The reason why Fraunhofer diffraction is a special case can be seen from Figure 31. The further the circular wavefronts travel away from their point sources, the more they tend to resemble plane waves. So, at infinity we are superimposing not two circular waves, but rather two *plane* waves. This is a much easier situation to analyse.

Now you may argue that when you performed the diffraction experiments, you were *not* a long way from the diffracting object—in fact you held the diffracting object (the 35 mm transparency) very close to your eye. This is true, but it does not invalidate the argument. Why? Because the light from the slit had to pass through the lens of your eye, and this lens was focused on the distant light source. In other words, your eye was adjusted so that only plane waves of light were focused on to the retina (see Figure 32). So

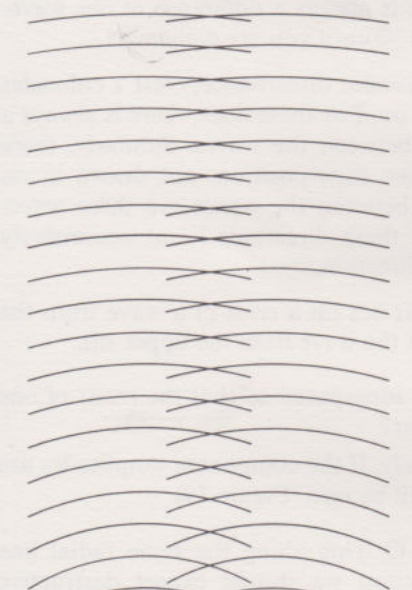


FIGURE 31 Circular wavefronts are produced by point sources. However, as the distance away from the sources increases, these wavefronts more and more closely resemble parallel *planes*.

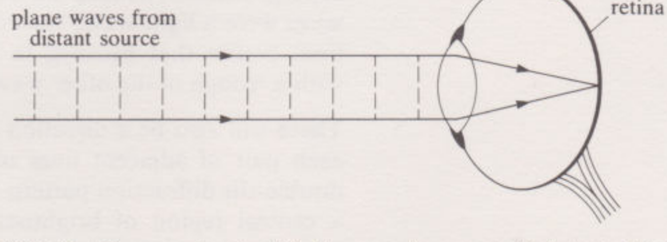


FIGURE 32 The wavefronts of light waves from a small distant object are effectively parallel planes by the time they reach the eye. This light can be brought to a focus—at the focal point of the eye lens—by the action of the convex lens of the eye.

the effect was *as if* the diffraction pattern had been produced at a large distance—your eye was focusing plane waves and *only* plane waves, and plane waves are what come from a small, very distant object (in the Experiment, the distant object was the filament of the candle bulb). We shall now demonstrate how these ideas help us to analyse mathematically the two-slit diffraction effect.

4.3.3 THE MATHEMATICS OF TWO-SLIT DIFFRACTION

Consider, just once more, two narrow slits separated by a distance d (Figure 33). If we suppose that the wave incident normally (at 90°) on the double-slit is a plane wave, then the waves emerging from the slit apertures must be exactly in phase. Now, as the illustration in Figure 30 indicates, light will exit from the slits in all directions (i.e. the wavefronts are circular at this point). However, if we concentrate only on the diffraction pattern produced at infinity, we need only consider light leaving the two slits in parallel directions. In Figure 33a, this is the straight-ahead direction. These parallel waves will then strike a **convex lens** (your eye lens in the actual experiment) which will bring the light to a focus, so that superposition of the waves will occur. (A convex lens brings light to a focus, and the point at which *parallel* light is brought to a focus is termed the focal point of the lens.) In the straight-ahead direction, since the two waves begin in step, and since they then subsequently both travel the same distance to the lens, the waves must still be in step at the lens's focal point. There will be constructive superposition.

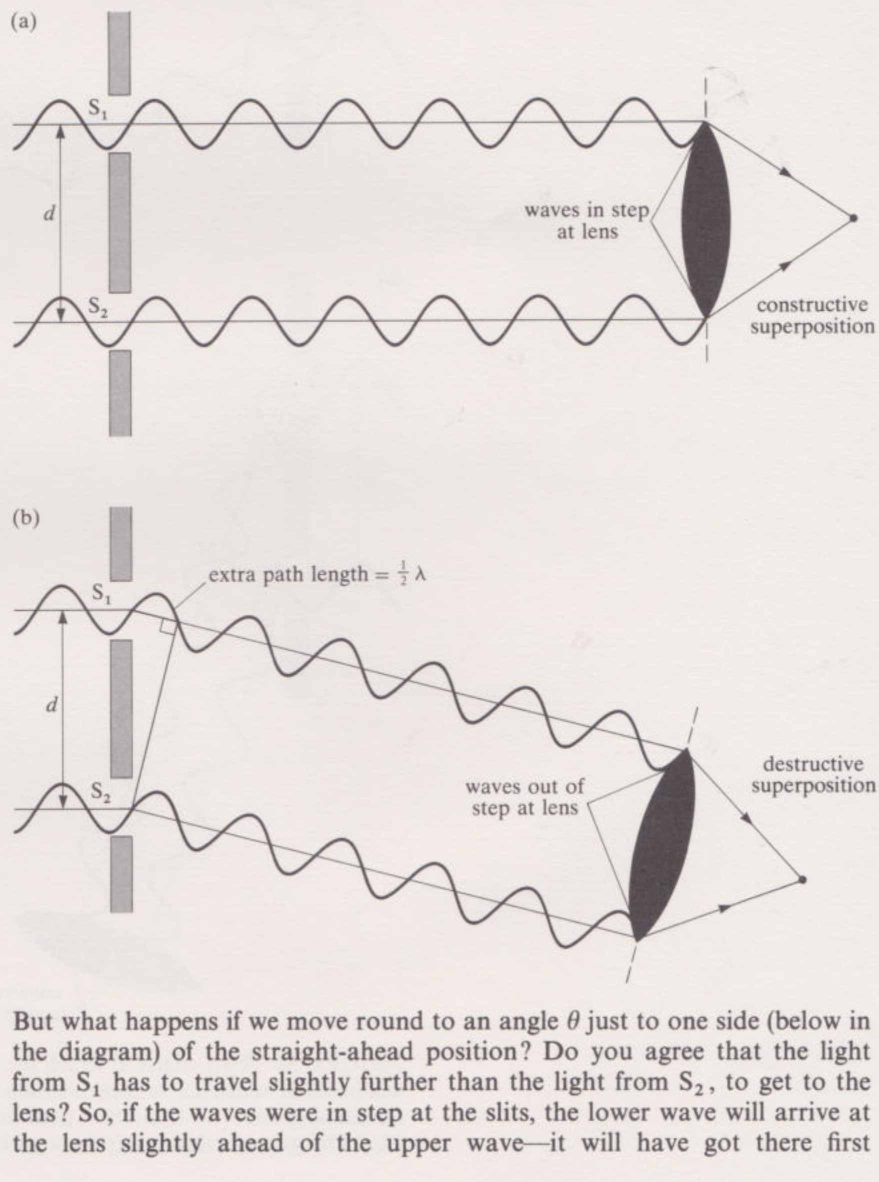


FIGURE 33 The superposition of waves from a double-slit, giving
(a) constructive superposition when the path difference is zero, and
(b) destructive superposition when the path difference is $\lambda/2$.

But what happens if we move round to an angle θ just to one side (below in the diagram) of the straight-ahead position? Do you agree that the light from S_1 has to travel slightly further than the light from S_2, to get to the lens? So, if the waves were in step at the slits, the lower wave will arrive at the lens slightly ahead of the upper wave—it will have got there first

DIFFRACTION EQUATION

because it didn't have quite so far to travel. The superposition at the lens's focal point won't now be perfectly constructive—the intensity will be less than in the straight-ahead direction.

If we move round to even larger angles, we eventually come to a direction in which the upper wave has to travel *exactly half a wavelength* further than the lower wave. Now the waves are exactly out of step at the lens, and we will get *destructive* superposition at the lens's focal point (Figure 33b).

Suppose we now move round to still larger values of the angle θ. The difference between the path lengths for the two waves will continue to increase, until it eventually becomes one whole wavelength (Figure 34a). But of course, when the path difference is a whole wavelength, the two waves will be in step again—there will be another region of constructive superposition at the lens's focal point. Let us call the angle at which this happens θ_1, as shown in Figure 34a. (Note that you can think of θ_1 as either the angular

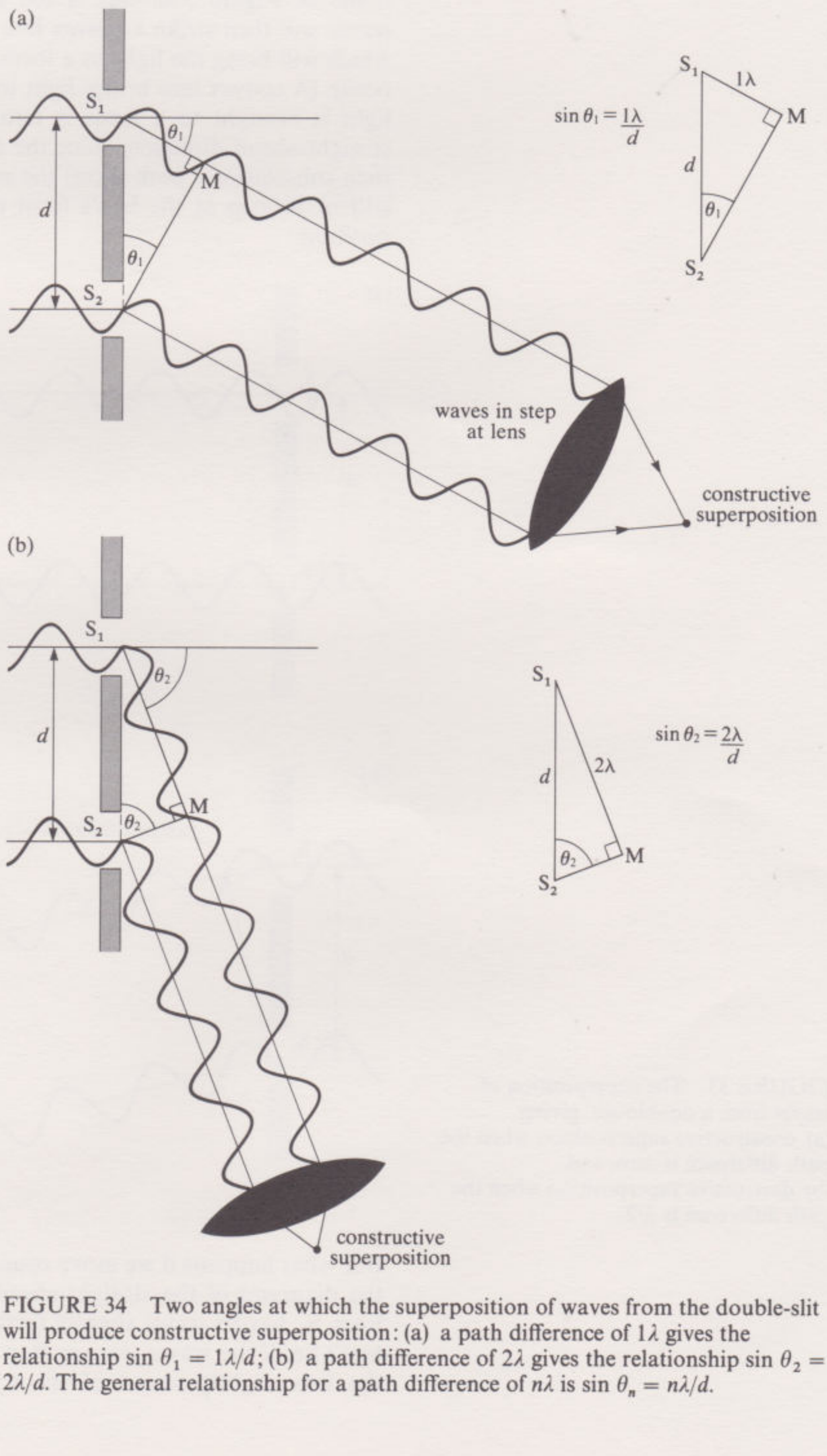


FIGURE 34 Two angles at which the superposition of waves from the double-slit will produce constructive superposition: (a) a path difference of 1λ gives the relationship $\sin \theta_1 = 1\lambda/d$; (b) a path difference of 2λ gives the relationship $\sin \theta_2 = 2\lambda/d$. The general relationship for a path difference of $n\lambda$ is $\sin \theta_n = n\lambda/d$.

change in direction from the straight-ahead direction, or as the angular change in the perpendicular to these directions; the two angles are both equal to θ_1.) As the inset diagram in Figure 34a shows, there is now a right-angled-triangle relationship between the angle θ_1, the path difference between the two waves (which in this case is exactly 1λ), and the separation of the double slits d. Since $\sin \theta_1$ is defined as the length of the side opposite θ_1 divided by the length of the hypotenuse in the triangle shown in Figure 34a,

$$\sin \theta_1 = 1\lambda/d$$

As the angle θ is increased further, there will be another region of destructive superposition (when the path difference is $3\lambda/2$), followed by another region of constructive superposition (when there are exactly two whole wavelengths difference between the path lengths travelled by the two waves—see Figure 34b). When the path difference is 2λ, let us call the corresponding angle θ_2. Then as you can see from Figure 34b,

$$\sin \theta_2 = 2\lambda/d$$

We can continue the argument, for still larger angles, until we simply run out of angular space (at $\theta = 90°$). In general, we will get constructive superposition whenever the difference in path lengths for the two waves to reach the lens is a *whole number* of wavelengths. We can write down this most general case mathematically as

$$\sin \theta_n = n\lambda/d$$

where n is any whole number. After multiplying both sides of this equation by d, we obtain the **diffraction equation**:

$$\boxed{d \sin \theta_n = n\lambda} \qquad (2)$$

This important equation tells us the angles (relative to the straight-ahead position) of each of the diffraction maxima, in terms of the spacing d of the double-slit and the wavelength λ of the incident light. Note that the values of $\sin \theta_n$ (and hence of θ_n) depend on λ and d. If λ or d is changed, the values of $\sin \theta_n$ and of θ_n will be different.

ITQ 6 A double-slit is illuminated with green light (as in your experiment). The diffraction pattern has a bright region of green light in the straight-ahead direction, with further bright regions positioned symmetrically on either side. The first two bright areas on either side occur respectively at angles of 0.40° and 0.80° to the straight-ahead direction.

(a) If the separation of the two slits is 0.08 mm, what is the wavelength of the green light, in units of nanometres, nm? ($1 \text{ nm} = 10^{-9} \text{ m}$.)

(b) Calculate (in nanometres) the difference in path length of the two waves that produce the constructive superposition generating the *second* bright region to the right of the straight-ahead direction.

4.4 MULTIPLE-SLIT DIFFRACTION: GRATINGS

4.4.1 THE CONSTRUCTIVE SUPERPOSITION CONDITION

We have already said that a diffraction grating is simply a large number of identical, equally-spaced parallel slits. To keep the arguments here comparable with the two-slit analysis, let us assume that the equal spacing between the slits is d, and that the width of each slit is extremely narrow. Suppose that we illuminate the grating with plane waves just as in the two-slit case. Then, as before, circular waves will emerge from each slit, and these waves will superpose to produce a resultant wave. Now there are not just two waves to superpose (as there were in the two-slit case), there are a very large number of waves to superpose. How will this change things?

DIFFRACTION ORDER

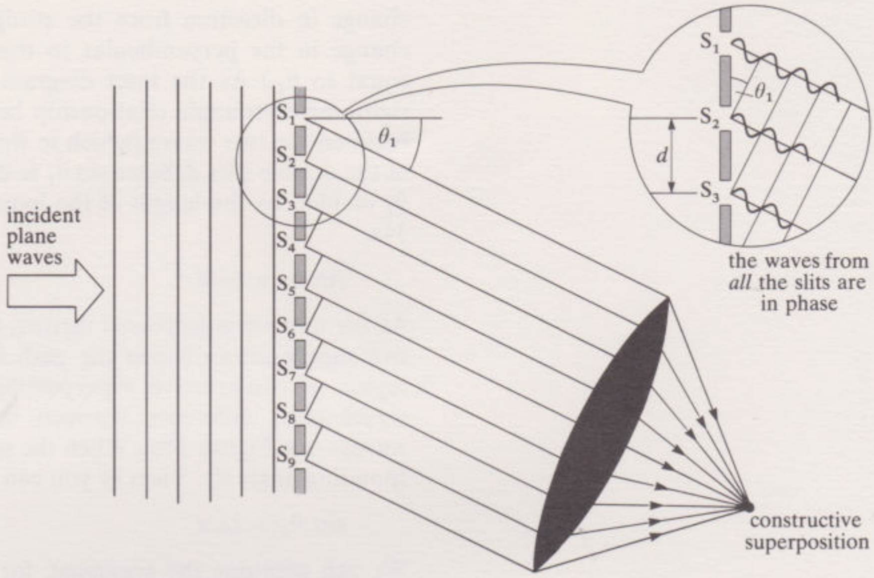


FIGURE 35 Diffraction by a grating. (For simplicity only nine slits have been shown.) At the angle θ_1 (the particular value shown in the Figure for the angle θ), the waves from each of the slits are in phase at the lens; consequently, constructive superposition occurs at this angle. The general equation for constructive superposition is once again $d \sin \theta_n = n\lambda$, where n is called the order of diffraction.

Look at Figure 35. In this diagram, the angle θ has been adjusted so that the path difference between the wave from S_1 and that from S_2 is exactly one wavelength; consequently, these two waves will be in phase at the lens, and will superpose *constructively*. But now look carefully at the path difference between the waves from S_2 and S_3—it's also exactly one wavelength. And the S_3–S_4 path difference, the S_4–S_5 path difference, the S_5–S_6 path difference, and so on right up to S_8–S_9, are all also exactly one wavelength. In other words, waves from any two *adjacent* slits differ by just *one* wavelength, are in phase at the lens, and will give constructive superposition. The angle θ_1 at which such superposition occurs is found in exactly the same way as for the double-slit case (Figure 34a):

$$d \sin \theta_1 = 1\lambda$$

We call this the *first order* diffraction equation.

Notice that when the path difference between waves from *adjacent* slits is exactly *one* wavelength, then the path difference between waves from *any* two slits is a *whole number* of wavelengths (see details in Figure 35). Thus each of the nine waves that emerge at an angle θ_1 will be in phase with all the other waves that emerge at this same angle.

As in the two-slit case, constructive superposition will occur again, at a larger angle θ_2, when the path difference between the waves from any two adjacent slits is two wavelengths (Figure 34b). The equation for this condition is

$$d \sin \theta_2 = 2\lambda$$

and this is said to be *second order diffraction.*

Clearly, there will be additional constructive superposition conditions when the path difference between the waves from adjacent slits is three wavelengths, or four wavelengths, or five wavelengths, etc.—in other words, when the path difference is n wavelengths (where n is any whole number). Hence, the general equation for constructive superposition is:

$$\boxed{d \sin \theta_n = n\lambda}$$

This is *exactly* the same equation as the diffraction equation for the double-slit (Equation 2). With diffraction gratings, however, n is usually called the **diffraction order**.

ITQ 7 At what angle relative to the straight-ahead direction would you expect to observe the zeroth order of diffraction?

4.4.2 THE DIFFRACTION GRATING PATTERN

What happens as we move to angles slightly away from a position at which constructive superposition occurs? With the double-slit, the waves from each slit began to get progressively out of phase with each other, resulting in gradual cancellation of the waves and hence lower light intensity, until eventually an angle was reached where the waves were completely out of phase, the superposition was completely destructive, and the light intensity was zero. With multiple-slits, essentially the same thing happens. But now, because there are *many* waves to get out of phase with each other, the fall-off in intensity is much more abrupt; when only a small way from a constructive superposition angle, the path differences between the very many component waves give rise to a resultant destructive-superposition condition. The more slits there are, the sharper (i.e. narrower) are the constructive superposition peaks. Figure 36 illustrates, both photographically and diagramatically, how the diffraction peaks get sharper as the number of slits increases.

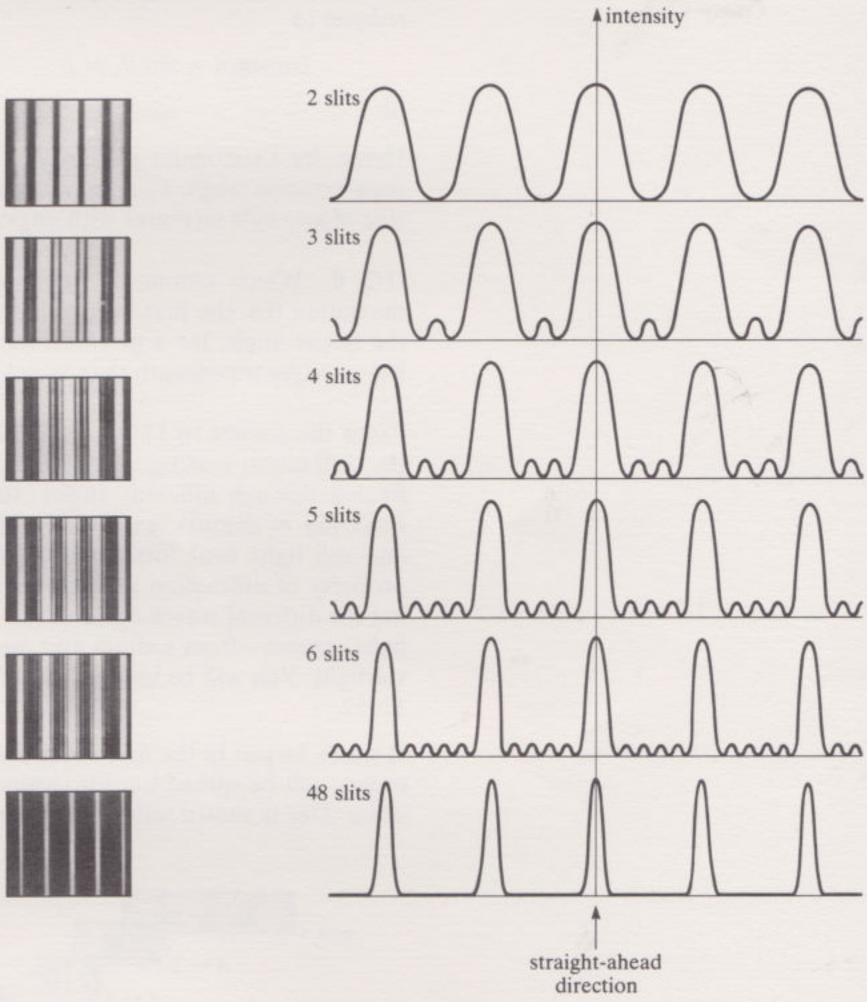


FIGURE 36 Multiple-slit diffraction patterns, obtained with light of a single wavelength. As the number of slits increases, the constructive superposition peaks become narrower. This implies that the angles at which constructive superposition occurs become much more sharply defined as the number of slits increases. (a) Photographs of the patterns; (b) graphical representations of the patterns (on an expanded horizontal scale).

4.4.3 DIFFRACTION GRATINGS AND COLOUR

In our discussion of diffraction so far, we have assumed that the incident light has a single wavelength (such light is said to be monochromatic). Light of a single wavelength has a particular colour: for example, red light has a wavelength of about 700 nm and violet light has a wavelength of about 400 nm (remember that $1\,\text{nm} = 10^{-9}\,\text{m}$). So we have been considering the diffraction of light of a single colour.

WHITE LIGHT

When light of a single wavelength is diffracted by a grating, we see a diffraction pattern like the one at the foot of Figure 36. That is, we see a straight-ahead zeroth order *line*, flanked on either side by *lines* of first order diffraction, second order diffraction, third order diffraction, etc. Visually, all these lines will be of the same colour—the colour of the incident light. This is what you saw in your experiment, when you placed the coloured filter in contact with your 35 mm transparency and then looked through them at the light source.

But what happens when **white light** is shone on the diffraction grating? White light is a combination of all the different colours of the visible spectrum (all the 'colours of the rainbow'), ranging from dark red light with a typical wavelength of about 700 nm, down to deep violet light with a wavelength in the region of 400 nm. But the diffraction equation $d \sin \theta_n = n\lambda$ actually contains the wavelength λ as one of the variables that can change. For a particular grating, d is fixed (i.e. d is constant). Thus if we restrict ourselves to observing only the first order ($n = 1$) diffraction, Equation 2 reduces to

$$\text{constant} \times \sin \theta_1 = \lambda$$

or

$$\sin \theta_1 \propto \lambda$$

Hence, for a particular grating, as λ increases, so the sine of the constructive superposition angle θ_1 also increases and therefore θ_1 increases (since the sine of an angle increases with angle, when the angle is between 0° and 90°).

ITQ 8 Which colour of light, red or violet, will have its first order maximum (i.e. the first bright line away from the centre of the pattern) at the larger angle, for a given diffraction grating? (Remember that red light has a longer wavelength than violet light.)

From the answer to ITQ 8, it must follow that if white light is incident on the diffraction grating, the different colours in the white light will be diffracted through different angles: the white light will be spread out into a spectrum of colours, with violet light nearest the straight-ahead direction, and red light bent furthest away from the straight-ahead direction. This property of diffraction gratings makes them invaluable tools for determining the different wavelengths of the various colours of light present in many light sources—from sodium and mercury street lamps, to sunlight and even starlight. You will be making use of this property in an experiment in Units 11–12.

It won't be just in the first order that a spectrum is produced—all the other orders will be spread out into spectra as well. All, that is, *except the zeroth order*. This is shown schematically in Figure 37.

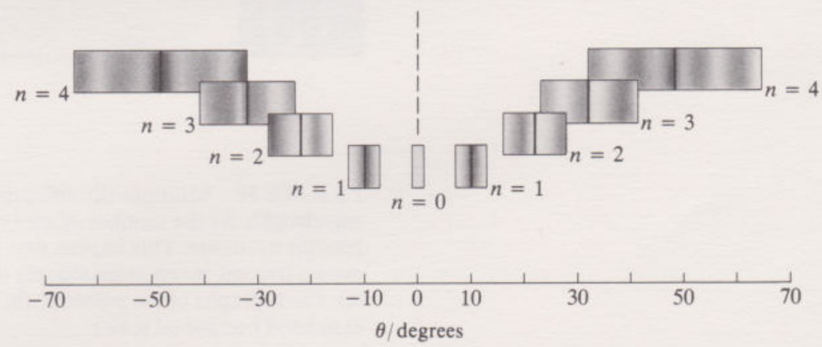


FIGURE 37 When a grating diffracts white light, a spectrum of colours is produced at each and every diffraction order *except the zeroth* (the straight-through line is not spread out). The vertical displacement shown in the diagram does not actually occur; it is used here for clarity, since, for the spectrum of white light, orders above the second would, in practice, overlap.

ITQ 9 Why is the zeroth order not spread out, and what colour will this order be?

SUMMARY OF SECTION 4

1 Huygens' construction enables us to predict the propagation of a wavefront. It says that every point on a wavefront can be thought of as a source of circular (or spherical) secondary wavefronts. The secondary waves have the same speed and wavelength as the original wave.

2 Huygens' construction provides an easy-to-visualize way of understanding (qualitatively) the phenomenon of diffraction.

3 Huygens' construction, combined with a mathematical analysis using calculus, can explain the diffraction pattern produced by a single-slit. The conclusions of this analysis are that the single-slit produces an alternating dark–bright diffraction pattern, spread out in the direction of the width of the slit. The *narrower* the slit, the *greater* the extent of each region of brightness.

4 Superposition of the (secondary) waves from a double-slit produces directions of maximum disturbance and directions of weak, or negligible, disturbance. The straight-ahead direction will always be one in which strong constructive superposition occurs. The angular separation of neighbouring directions of constructive superposition will depend on the separation between the two slits: widely spaced slits will produce a small angular separation between directions of constructive superposition; closely spaced slits will produce a wide angular separation. There will be directions of destructive superposition midway between adjacent regions of strong constructive superposition.

5 The diffraction pattern produced a long distance away from the diffracting object is known as the Fraunhofer diffraction pattern. If a convex lens is placed so as to intercept the waves from the diffracting object, the Fraunhofer diffraction pattern will be seen at the focus of the lens.

6 The Fraunhofer diffraction pattern of a double-slit comprises a series of almost equally-spaced regions, in which intensity changes smoothly (with position) from bright to dark to bright to dark, etc.

7 Directions of strong constructive superposition in the double-slit diffraction pattern always occur when there is a path difference between the waves from the two slits of a whole number of wavelengths. The angles θ_n (relative to the straight-ahead direction) at which constructive superposition occurs can be calculated from the diffraction equation,

$$d \sin \theta_n = n\lambda$$

where d is the separation of the slits, λ is the wavelength of the incident wave, and n is any whole number.

8 A diffraction grating is an array of a large number of equally-spaced parallel slits. When light of wavelength λ is diffracted by a grating, the angles θ_n at which constructive superposition occurs are specified by the same diffraction equation as for the double-slit:

$$d \sin \theta_n = n\lambda$$

For a diffraction grating, n is known as the diffraction order.

9 The Fraunhofer diffraction pattern of a grating with a slit spacing d, is the same as that for a double-slit of separation d, except that the bright regions are now narrowed down to sharp lines. The more slits there are in the grating, the sharper and brighter are the peaks of brightness in the diffraction pattern.

10 For a particular diffraction grating (d = constant), used in a particular diffraction order (n = constant), $\sin \theta_n$ is proportional to λ. Hence the grating can be used to separate the different colours (wavelengths) present in a light source.

SPEED OF LIGHT IN A VACUUM

ELECTROMAGNETIC WAVES

VISIBLE LIGHT

SAQ 4 The adjacent slits on a particular diffraction grating are separated by 2×10^{-6} m. A beam of laser light is incident perpendicularly on the grating and is diffracted in such a way that a *second* order diffraction line is at an angle of 30° to the straight-through direction.

(a) What is the wavelength λ of the laser light, in units of nanometres?

(b) At what angle will the *first* order diffraction line be observed?

SAQ 5 With another diffraction grating, the light from a particular (distant) light source is diffracted so that a second-order diffracted orange line of wavelength 600 nm exactly coincides with a third-order violet line (i.e. the second-order orange light and the third-order violet light are diffracted through the same angle). What is the wavelength of the violet light?

SAQ 6 Parallel water waves with a wavelength λ of 10 mm are incident on a double-slit obstacle in a ripple tank (Figure 38). The (very narrow) slits S_1 and S_2 are separated by a distance d of 35 mm. Beyond the double slit there appear regions of constructive superposition, regions of destructive superposition, and regions of intermediate superposition. Calculate which type of superposition occurs at the two positions P_1 and P_2 shown in Figure 38.

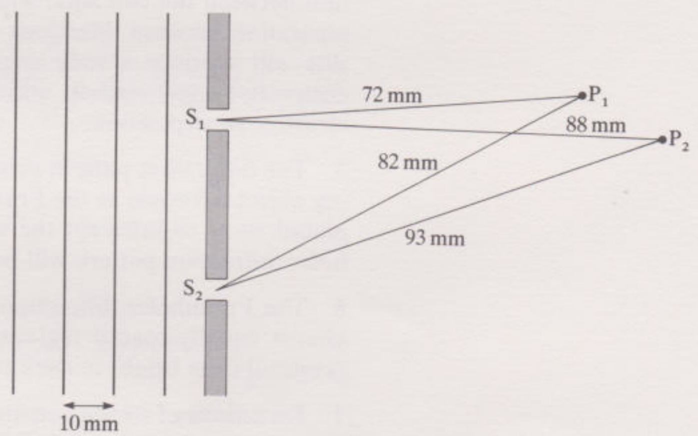


FIGURE 38 For use with SAQ 6.

SAQ 7 For the double-slit example in SAQ 6, at what angles (relative to the straight-ahead direction) will constructive superposition occur?

5 WHAT KIND OF WAVE?

It's rather remarkable that we have been able to explain so much about the diffraction effects exhibited by light, solely on the assumption that 'light behaves as a wave'. Nowhere in our discussion of diffraction did we have to specify the precise nature of the light wave; indeed, our conclusions were reached, essentially, by analogy with water waves! What was important was the fact that light's behaviour is *wave-like*.

In this Section, we shall describe the nature of light waves. As you will see, they are a type of electromagnetic wave. But first, we must consider the speed at which light travels.

5.1 THE SPEED OF LIGHT

The speed of light depends on the medium in which it travels: for example, its speed in air (in normal atmospheric conditions) is about 33% greater than its speed in water. Experiments have shown that the speed of light is greatest when it travels in no medium at all, that is, when it travels in a vacuum. The **speed of light in a vacuum**, which is denoted by the letter c, is

about 300 million metres per second;* that's about a million times the speed of a bullet from a gun. (The speed of light in air is slightly less than its speed in a vacuum, but for the purposes of this Course, we can ignore the difference between the two speeds.)

Using the relationship given in the AV sequence between the speed, frequency and wavelength of a wave (speed = frequency × wavelength), it follows that for light travelling in a vacuum,

$$c = f\lambda \tag{3}$$

where f is the frequency of the light and λ is its wavelength. For example, consider green light. This light has a wavelength λ_{green} of about 560 nm, that is 5.6×10^{-7} m, so Equation 3 tells us that its frequency f_{green} is

$$\begin{aligned} f_{\text{green}} &= \frac{c}{\lambda_{\text{green}}} \\ &= \frac{3.0 \times 10^{8}\,\text{m}\,\text{s}^{-1}}{5.6 \times 10^{-7}\,\text{m}} \\ &\approx 5.4 \times 10^{14}\,\text{Hz} \end{aligned}$$

5.2 ELECTROMAGNETIC WAVES

In 1865, the Scottish theoretical physicist James Clerk Maxwell published a set of equations that embodied his unified theory of electricity and magnetism. He later used this brilliant theory to predict the very nature of light—he predicted that light is a type of **electromagnetic wave**. The German physicist Heinrich Hertz (after whom the unit of frequency is named) later demonstrated the validity of Maxwell's prediction, but alas Maxwell died seven years before this triumphant vindication of his theory.

How can an electromagnetic wave be visualized? Well, in Figure 39 we've shown a 'snapshot' of this type of wave travelling in a vacuum. As this Figure shows, the wave is really *two* sinusoidal waves at right angles to each other. One wave is an oscillating magnetic field, the other an oscillating electric field;† each has the same wavelength, which is of course the wavelength λ of the light. The two components are in phase with one another and they remain in phase as they propagate at the speed of light.

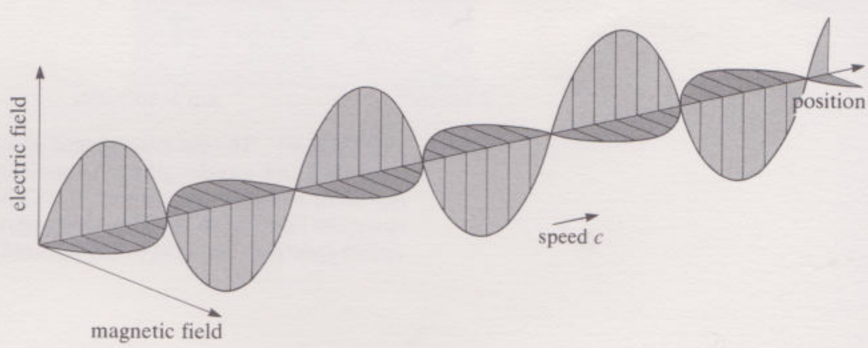


FIGURE 39 An electromagnetic wave travelling in a vacuum.

The wavelengths of **visible light**, which we can easily measure in diffraction experiments, lie in the range 400–700 nm. But there are no theoretical reasons why electromagnetic waves should not be able to have wavelengths *outside* this range. Surely, the wavelength of an electromagnetic wave can be of *any* length? It was this argument that led Maxwell to predict correctly the existence of other types of electromagnetic wave.

* You may remember from Unit 2 that the speed of light in a vacuum, c, is now *defined* to be *exactly* $2.997\,924\,58 \times 10^{8}\,\text{m}\,\text{s}^{-1}$. However, for most purposes in this Course, it is sufficiently accurate if you take the value of c to be $3.00 \times 10^{8}\,\text{m}\,\text{s}^{-1}$.

† The electric field at a point is defined as the electrostatic force (at the point) per unit positive charge.

ELECTROMAGNETIC RADIATION

ELECTROMAGNETIC SPECTRUM

We now know that there are types of electromagnetic wave that are *not* visible to the human eye but that travel in a vacuum at the same speed c as light: Equation 3 ($c = f\lambda$) applies to *all* electromagnetic waves, not just to light. Hence, it would perhaps be better to refer to c as the speed of electromagnetic waves in a vacuum. However, that terminology has not caught on, so we shall not use it in this Course.

Figure 40 shows the various types of electromagnetic wave, or as they are sometimes called, types of **electromagnetic radiation**. As you can see from Figure 40, the **electromagnetic spectrum** ranges from the short-wavelength extreme of γ-rays (pronounced 'gamma rays'), which have wavelengths below about 10^{-11} m, to the long-wavelength extreme of radio waves, which have wavelengths greater than about 10^{-1} m. It is worth stressing that all these types of radiation are similar, in the sense that they all travel in a vacuum at the same speed and each can be represented by Figure 39; the only difference between them is that their wavelengths (and therefore their frequencies) are different. Note that although waves in different parts of the spectrum are given names, the boundaries are not sharply defined.

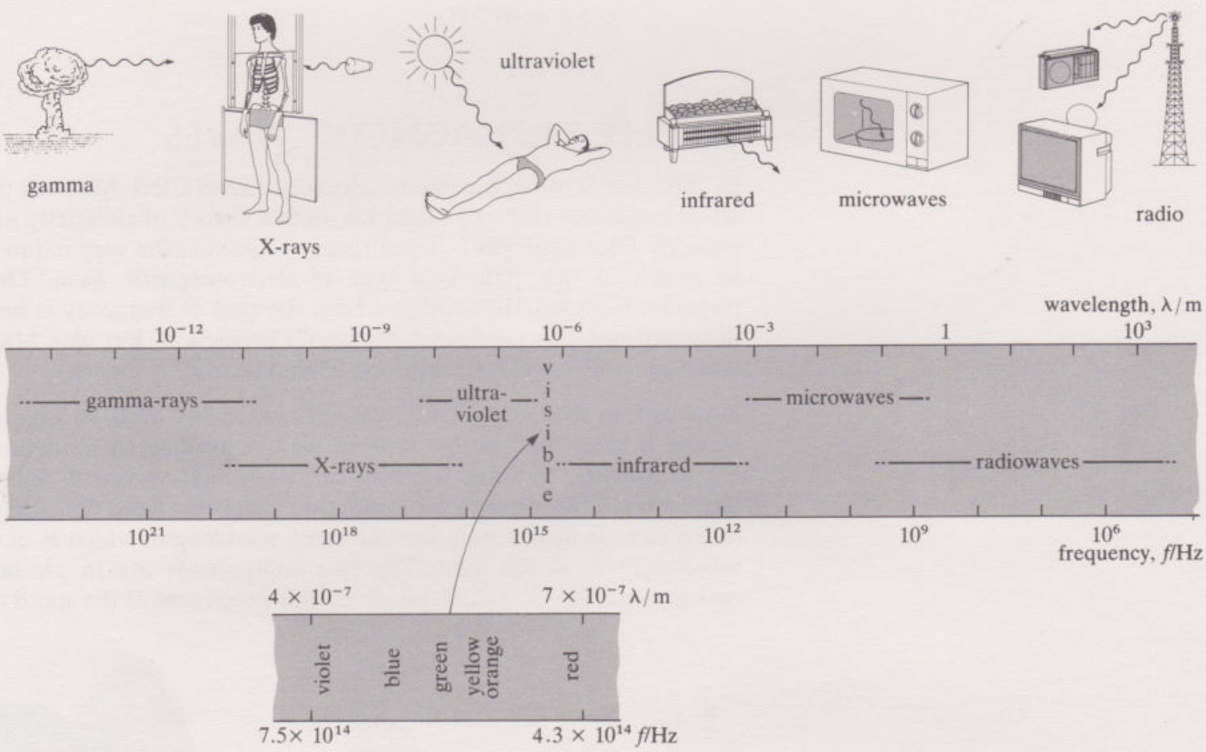


FIGURE 40 The electromagnetic spectrum. Note that the scales used for frequency and wavelengths are logarithmic (i.e. the equal intervals on the scale represent a change by a power of ten). Note also that the violet end of the visible spectrum lies next to the ultraviolet part of the spectrum, whereas the red end of the visible spectrum lies next to the infrared part of the spectrum.

How are electromagnetic waves generated? This question can be answered simply, by analogy with the generation of water waves. Just as you can produce waves in a ripple tank or a bath by waggling your hand up and down in the water, so electromagnetic waves can be produced by an oscillating charged particle, such as an electron (Figure 41). For example, it is the motion of the electrons in the filament of a light bulb that gives rise to the emitted light. Similarly, the microwave radiation in a microwave oven is produced by the motion of electrons (in a device known as a magnetron).

It is surely unlikely that Maxwell could have foreseen the impact of the applications of electromagnetic radiation on human life. What would he have made of X-ray machines, sun-lamps, radios, TVs, microwave ovens

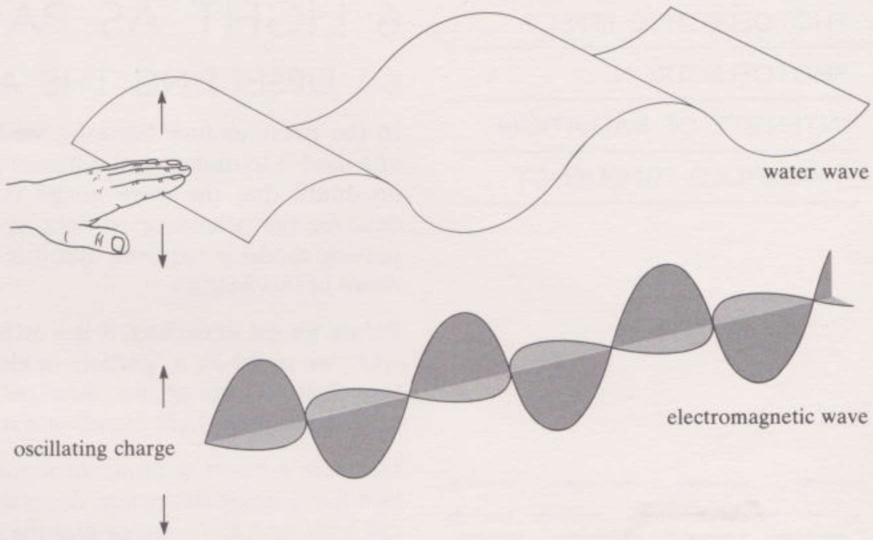


FIGURE 41 By moving your hand up and down in water, you can generate a water wave. Similarly, if an electrical charge oscillates, an electromagnetic wave is generated.

and compact disc players? The operation of each of these devices depends on parts of the electromagnetic spectrum that cannot be seen by the human eye—but which Maxwell was the first to see theoretically with the aid of his equations of electricity and magnetism.

SUMMARY OF SECTION 5

1 Each electromagnetic wave has two components—a sinusoidally varying electric field and a sinusoidally varying magnetic field (Figure 39).

2 According to Maxwell's ideas, visible light is simply an electromagnetic wave with a wavelength in the range 400–700 nm.

3 There exists a spectrum of electromagnetic waves, ranging from γ-rays to radio waves. This is called the electromagnetic spectrum (Figure 40).

4 All electromagnetic waves travel in a vacuum at the same speed c, which is $3.00 \times 10^8\,\mathrm{m\,s^{-1}}$ (to three significant figures). The relationship $c = f\lambda$ therefore applies to all electromagnetic waves.

SAQ 8 A radio station broadcasts using waves that have a frequency of 22 kHz. What is the wavelength of these waves?

SAQ 9 Most domestic microwave ovens use microwave radiation that has a frequency of 2.45×10^9 Hz.

(a) What is the wavelength of the wave?

(b) Sketch a 'snapshot' of a microwave in a microwave oven, drawing it to its *actual size*.

SAQ 10 In a compact disc player, information is read from the rotating disc using a laser beam of electromagnetic radiation that has a wavelength of approximately 800 nm. Is this radiation visible to the human eye?

PHOTOELECTRIC EFFECT

PHOTOELECTRON

INTENSITY OF RADIATION

THRESHOLD FREQUENCY

6 LIGHT AS PARTICLES

6.1 UPSETTING THE APPLECART

In the previous four Sections, we have described the wave model of light and used it to understand different phenomena, notably diffraction. There is no doubt that the wave model is extremely useful but, alas, it does not describe the behaviour of light in *all* circumstances. In some situations a particle model is required, and it is these situations that we shall consider in detail in this Section.

Before we get underway, it is worth pausing for a moment to specify clearly what we mean by a 'particle model of light'. In this model, it is supposed that *light consists of tiny, localized particles, each of which has energy and momentum. If the light travels in a vacuum, each particle moves with a speed c.*

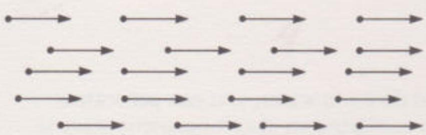


FIGURE 42 A beam of light, visualized using a particle model of light.

Consider a beam of monochromatic red light (Figure 42), which by definition has a single frequency. According to the particle model, the particles of red light each have energy and the direction of their momentum is the same as the direction of the beam (remember from Units 3 and 9 that momentum is a quantity that has a magnitude *and* direction). Note that the particle model applies not only to light but to all types of electromagnetic radiation—this Section should perhaps have been called 'Electromagnetic radiation as particles', but that would have been rather clumsy!

It was Einstein who in 1905 showed that the *energy* of radiation is delivered in discrete amounts, but it was nearly twenty years before it was demonstrated that these 'bundles of energy' also have *momentum*. Only then was the particle model of radiation fully established. The particles of electromagnetic radiation are now called photons (from the Greek word *photos*, meaning light).

6.2 THE PHOTOELECTRIC EFFECT

6.2.1 WHAT IS THE PHOTOELECTRIC EFFECT?

Under the right conditions, when electromagnetic radiation is shone onto a solid, electrons are ejected with a range of kinetic energies up to a well-defined maximum value. This is the **photoelectric effect** (Figure 43). The ejected electrons are normally called **photoelectrons** (but note that electrons and photoelectrons are not different types of particle—on the contrary, they are the same particle with different names).

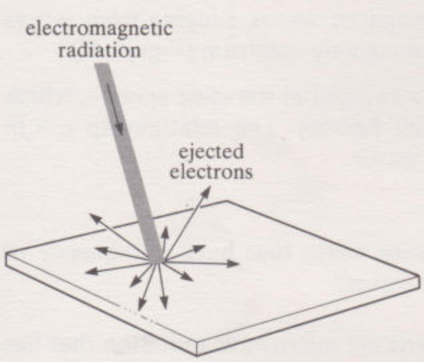


FIGURE 43 The photoelectric effect.

The photoelectric effect shows that solids contain electrons, but what does it tell us about electromagnetic radiation? Before we answer this central question, it's helpful to look in detail at a specific example of the effect: an experiment in which radiation is shone onto a piece of iron (Figures 44 and 45).

The first point to make is that photoelectrons are ejected at the instant that radiation impinges on the iron. In other words, there is *no time delay* between the instant that the radiation strikes the solid and the instant that the photoelectrons are ejected.

Now let's see what happens when a beam of radiation with a particular frequency, 5×10^{15} Hz, is shone onto the piece of iron. It is found experimentally that the maximum kinetic energy of the ejected electrons is 16.1 eV. (Note that we are using the electronvolt, eV, as our unit of energy here—as we mentioned in Unit 9, it is often more convenient to use this unit in atomic science.)

If the frequency of the incident radiation is increased, it is found that the maximum kinetic energy $(E_k)_{max}$ of the ejected electrons increases; similarly, if the frequency of the radiation is decreased the electrons' maximum kinetic energy decreases. However—and this is a very important point—*when the frequency of the radiation is below 1.1×10^{15} Hz, no electrons at all are ejected.*

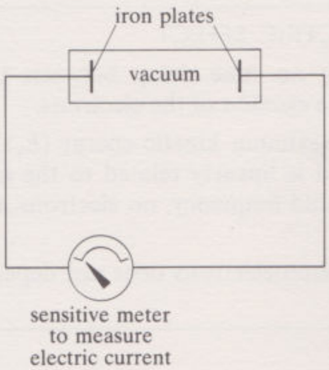


FIGURE 44 Before the radiation is shone on the iron, no current flows in the circuit (which is broken by the vacuum in the container).

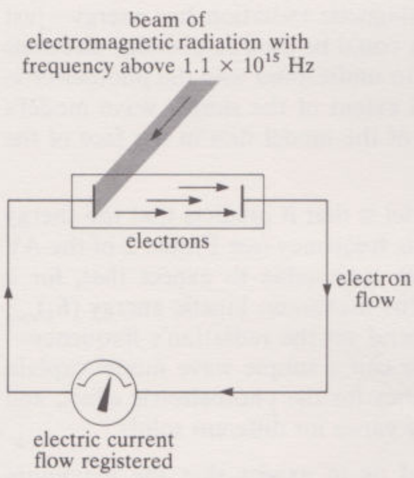


FIGURE 45 When radiation that has a frequency above 1.1×10^{15} Hz impinges on one of the plates, electrons are ejected from the iron and so an electric current flows in the circuit.

ITQ 10 Which *one* of the following types of electromagnetic radiation would eject electrons from the piece of iron?

(a) infrared

(b) microwaves

(c) radio waves

(d) X-rays

(Hint: Refer to the electromagnetic spectrum, shown both in Figure 40 and on the back cover of the Unit.)

The maximum kinetic energy of the photoelectrons ejected from iron is plotted against the frequency of the incident radiation in Figure 46. This shows that *the maximum kinetic energy* $(E_k)_{max}$ *of the photoelectrons ejected from iron depends linearly on the radiation's frequency.* But how does the photoelectrons' maximum kinetic energy depend on the radiation's intensity? The **intensity** of a beam of electromagnetic radiation is defined as the amount of energy carried by the beam in unit time across unit area perpendicular to the beam's direction of motion.

You might expect that the higher the intensity of the radiation incident on the iron, the higher would be the maximum kinetic energy. After all, isn't it reasonable to expect that the electrons will be ejected from the iron with a much higher kinetic energy if the incident radiation is emitted from a high-powered laser than if the radiation is much less intense? In fact, it turns out that *the maximum kinetic energy of the photoelectrons does not depend in any way upon the radiation's intensity.*

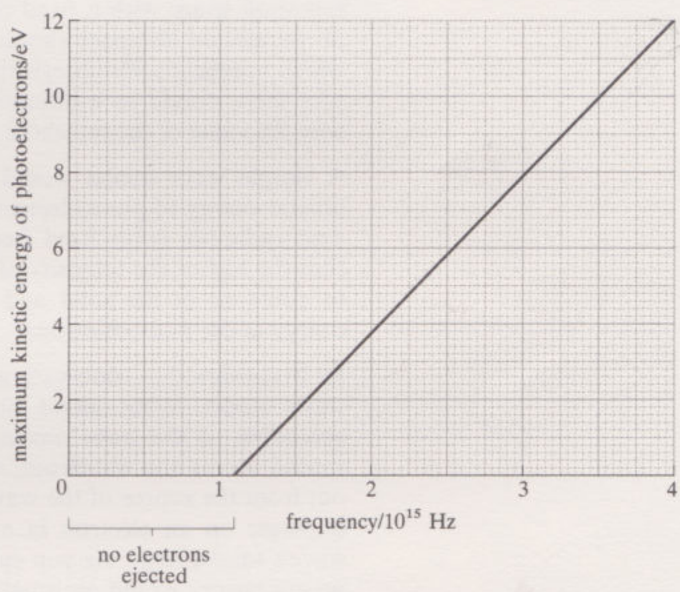


FIGURE 46 Data from experiments show that there is a linear relationship between the maximum kinetic energy of the photoelectrons ejected from iron and the frequency of the incident radiation.

Photoelectric experiments with other solids lead to the same general conclusions as those with iron. The only difference is that each solid has its own characteristic **threshold frequency**—when the incident radiation's frequency is *below* this threshold, no electrons are ejected, no matter how high the intensity of the radiation. Thus 1.1×10^{15} Hz (Figure 46) is known as the threshold frequency for iron.

A summary of the key points about experimental observations of the photoelectric effect is given over the page.

QUANTIZATION

PLANCK'S CONSTANT h

OBSERVATIONS OF THE PHOTOELECTRIC EFFECT

1 When electrons are ejected, there is no time delay between the impinging of radiation on the solid and the ejection of the electrons.

2 Above the threshold frequency, the maximum kinetic energy $(E_k)_{max}$ of the photoelectrons ejected from a solid is linearly related to the frequency of the radiation. Below the threshold frequency, no electrons are ejected.

3 The maximum kinetic energy of the photoelectrons does not depend on the intensity of the radiation.

6.2.2 A FAILURE OF THE WAVE MODEL

How closely do these experimental observations compare with expectations based on a simple wave model? The answer is: very poorly indeed! Let's see why.

According to the wave model, electromagnetic radiation has energy—just like any other type of wave. This energy could be used to dislodge electrons from a solid, so the model can be used to understand *why* the photoelectric effect occurs. That, however, is the full extent of the simple wave model's success: from now on, each prediction of the model flies in the face of the experimental observations.

The central problem with the wave model is that it predicts that the energy of the radiation should *not* depend on its frequency (see Frame 2 of the AV sequence in Section 3.1). It is therefore reasonable to expect that, for a radiation beam with a fixed intensity, the maximum kinetic energy $(E_k)_{max}$ of the ejected electrons should not depend on the radiation's frequency—which conflicts with observation 2. Nor can a simple wave model explain why there should be a threshold frequency for the photoelectric effect, and why the value of the threshold frequency varies for different solids.

A simple wave model would also lead us to expect that the maximum kinetic energy of photoelectrons would depend on the intensity of the incident radiation (with fixed frequency). This is because the model predicts that the higher the intensity of the radiation, the more energy is transferred to electrons in the solid and therefore the greater the maximum kinetic energy of the photoelectrons.

So it appears that observations 2 and 3 cannot be explained by the simple wave model. What about observation 1, concerning time delay? Well, according to the wave model, the energy of the electromagnetic wave is spread across the wavefront, whose area gradually increases as it spreads out from the source of the wave. Hence, the tiny area of the wavefront that impinges on an electron in a solid will have only a tiny fraction of the wave's total energy. As you can probably deduce intuitively, enough of the wave's energy would eventually 'accumulate' to eject an electron. Detailed calculations with a simple wave model show that under normal laboratory conditions (using an ordinary 100 W radiation source, a few centimetres from the solid) the time delay should be of the order of a few minutes. This flatly contradicts observation 1, according to which there is no time delay at all!

So the simple wave model cannot explain the details of the photoelectric effect. How does a particle model solve these problems?

6.2.3 EINSTEIN'S THEORY OF THE PHOTOELECTRIC EFFECT

In 1902, Albert Einstein was appointed to the patent office in Bern, Switzerland, as a technical expert (third class). In his spare time, both inside and outside the office, he worked on some fundamental scientific problems. This work bore the most remarkable fruit: in 1905, he published three of the most influential of all scientific papers. In one he formulated the special

theory of relativity, in another he put forward a theory that convinced almost all scientists of the existence of atoms, and in the third he explained the photoelectric effect. It is perhaps particularly remarkable that during this extraordinarily productive time of his life, he and his wife were looking after their first child, then a one-year-old baby!

In this Section, we shall be concerned solely with his paper on the photoelectric effect, in which he put forward a theory of electromagnetic radiation. The essence of the theory is that the energy of radiation is **quantized**—it is absorbed and emitted in discrete amounts called *quanta*. For electromagnetic radiation with frequency f, the kinetic energy of each quantum is given by

$$\boxed{E = hf} \tag{4}$$

where h is a universal constant now known as **Planck's constant**,* which is approximately equal to 6.63×10^{-34} J s. The crucial point to note here is that, according to Equation 4, the energy of radiation with a given frequency is not delivered continuously (like the energy of a wave) or in variable amounts—it is always delivered in quanta that each have exactly the same energy.

The tiny magnitude of Planck's constant implies that the energy of radiation of each quantum is, by everyday standards, very small indeed.

□ What is the energy of each quantum of microwave radiation that has a frequency of 2.45×10^9 Hz?

■ 1.6×10^{-24} J—a tiny fraction of a joule! This energy is easily calculated from Equation 4, $E = hf$:

$$\begin{aligned}\text{energy of the microwave quantum} \\ &= (6.63 \times 10^{-34}\,\text{J s}) \times (2.45 \times 10^9\,\text{Hz}) \\ &\approx 1.6 \times 10^{-24}\,\text{J}\end{aligned}$$

(because, by definition, $1\,\text{Hz} = 1\,\text{s}^{-1}$). This is the energy of each quantum of microwave radiation in a domestic microwave oven.

The equation $E = hf$ is so important that it is well worthwhile for you to pause to check that you can use it. Try ITQ 11.

ITQ 11 (a) Use the electromagnetic spectrum (Figure 40, also repeated on the back cover of this book) to find to the nearest order of magnitude the energy of a quantum of visible light:

(i) in units of joules,

(ii) in units of electronvolts (the factor that enables you to convert from joules to electronvolts is given on the back cover).

(b) The total energy of the visible light emitted in one second by an ordinary domestic light bulb is about 10 J. To the nearest order of magnitude, how many quanta of visible light are emitted from such a bulb in one second?

How can the idea of radiation quanta be applied to the photoelectric effect? Einstein modelled the effect in terms of collisions between individual quanta and electrons. More specifically, he suggested that it could be assumed that each electron is ejected from the solid by a single quantum whose energy is

* Equation 4 was first written down in 1900 by the German physicist Max Planck, who is generally credited with the discovery of the quantization of the energy of electromagnetic radiation. However, there is little doubt that it was Einstein who first fully understood the implications of the idea.

WORK FUNCTION, ϕ

EINSTEIN'S PHOTOELECTRIC EQUATION

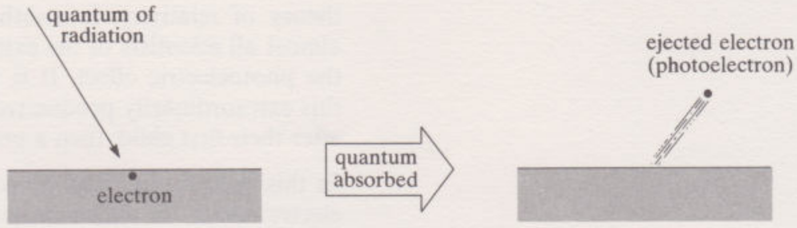


FIGURE 47 In the photoelectric effect, each ejected electron (photoelectron) is ejected by absorbing all the energy of a single quantum of radiation.

entirely absorbed (Figure 47). Because the collision between the electron and the incoming radiation quantum takes place at a point, there should be no time delay between the collision and the ejection of the electron; they should both happen at the same instant. So observation 1 is easily understood in terms of Einstein's theory.

Now what about observations 2 and 3, which concerned the photoelectrons' maximum kinetic energy? Einstein explained these observations by applying the law of conservation of energy to a single collision between a quantum of radiation and an electron in the solid. We shall now show how he did this—you'll probably be surprised by how easy it is!

TABLE 1 Work functions of eight solids

Solid	Work function/eV
chromium	4.44
iron	4.60
lead	4.01
nickel	5.15
platinum	5.63
silver	4.44
tin	4.28
zinc	4.33

Consider a collision between a quantum of radiation of frequency f and an electron (in a solid) that is immediately ejected from the solid. Before the electron emerged it was bound in the solid. The electrons, however, are not all bound equally tightly, and this is reflected in the fact that it does not require the same energy to remove each electron from a solid. The least tightly bound electrons obviously require the smallest energy to eject them. The *smallest* energy that can remove an electron from a particular solid is called the solid's **work function, ϕ** (pronounced 'phi'). The values of the work functions for a number of solids are listed in Table 1.

According to the law of conservation of energy, the total energy after the collision is equal to the total energy before. Hence,

$$\begin{array}{c}\text{kinetic energy of}\\\text{ejected electron}\end{array} = \begin{array}{c}\text{energy of}\\\text{radiation}\\\text{quantum}\end{array} - \begin{array}{c}\text{energy required to}\\\text{remove the electron}\\\text{from the solid}\end{array} \qquad (5)$$

Less formally, you could say that part of the energy of the radiation quantum is used to pull the electron out of the solid, and the remainder becomes the photoelectron's energy of motion.

What is the maximum kinetic energy $(E_k)_{max}$ of the ejected photoelectrons? Well, it should be fairly obvious that when radiation of a given frequency f impinges on a solid, the photoelectrons ejected with the *maximum* kinetic energy $(E_k)_{max}$ were the *least* tightly bound electrons in the solid:

$$\begin{array}{c}\textit{maximum}\text{ kinetic}\\\text{energy }(E_k)_{max}\\\text{of photoelectrons}\end{array} = \begin{array}{c}\text{energy } hf \text{ of}\\\text{an incident}\\\text{radiation}\\\text{quantum}\end{array} - \begin{array}{c}\textit{minimum}\text{ energy to}\\\text{eject photoelectron}\\\text{from the solid (i.e.}\\\text{the solid's work}\\\text{function } \phi)\end{array}$$

This equation can be written in an exactly equivalent but more succinct form:

$$\boxed{(E_k)_{max} = hf - \phi} \qquad (6)$$

This is known as **Einstein's photoelectric equation**, and it is the expression we have been seeking for the maximum kinetic energy of the ejected electrons. This equation enables observations 2 and 3 to be explained very easily; but before we give the explanation, you should do ITQ 12. Don't be tempted to skip it!

ITQ 12 The work function of lead is approximately 4 eV (Table 1).

(a) What happens when a beam of ultraviolet radiation, whose radiation quanta each have an energy of 10 eV, is shone onto a piece of lead?

(b) Are electrons emitted when a beam of visible light, whose radiation quanta each have an energy of 2 eV, is shone onto a piece of lead?

It should now be clear that no electrons will be ejected if the energy hf of the incoming radiation quantum is less than the work function ϕ of the solid. This makes good sense: for an electron to be ejected, the energy of the radiation quantum must be greater than or equal to the minimum energy (ϕ) required to eject an electron.

If you look back to Equation 6, you will see that Einstein's photoelectric equation makes a very clear prediction about the dependence of the maximum kinetic energy $(E_k)_{max}$ of the photoelectrons on the frequency of the incident radiation. The equation says that there is a linear relationship between the two—the graph of $(E_k)_{max}$ plotted against frequency should be a straight line with gradient h and with an intercept on the f-axis of ϕ/h* (Figure 48). This prediction is completely in agreement with observation 2.

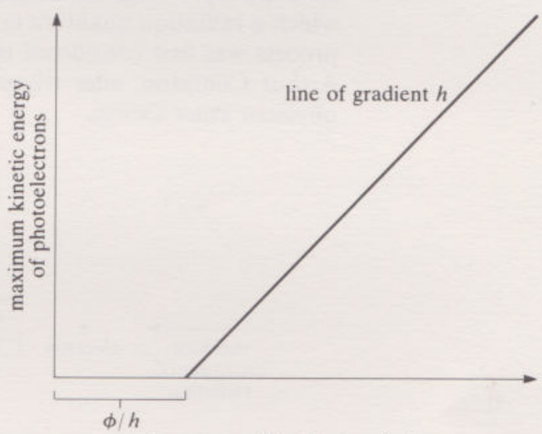


FIGURE 48 Einstein's theory of the photoelectric effect predicts correctly that there is a linear relationship between the maximum kinetic energy of ejected photoelectrons and the frequency of the incident radiation.

What about observation 3? This says that $(E_k)_{max}$ does not depend on the intensity of the incident radiation. Einstein's theory explains this easily: $(E_k)_{max}$ depends on the energy of each *quantum* of the radiation, which in turn is determined by the radiation's *frequency* ($E = hf$, Equation 4). According to Einstein, then, the intensity of the radiation does not in any way determine the maximum kinetic energy of the photoelectrons. (However, the theory does predict that, above the threshold frequency, the number of photoelectrons ejected is proportional to the radiation's intensity. This is borne out experimentally.)

So Einstein's theory of the photoelectric effect accounts beautifully for each of the observations 1–3, none of which can be explained by a simple wave model. The ingredients of the theory that are most crucial to its success are:

(i) the energy of radiation is quantized (i.e. the energy of radiation is delivered in discrete amounts); and

(ii) each quantum of radiation can be transferred to a single electron.

* These values of the gradient and intercept can be understood by noting that Equation 6, $(E_k)_{max} = hf - \phi$, is of the same type as the standard equation, $y = ax + b$: y corresponds to $(E_k)_{max}$, a to h, x to f, and b to $-\phi$. (You met the standard equation in *MAFS 3*: if y is plotted against x, the graph obtained is a straight line with gradient a, and its intercept on the x-axis, where $y = 0$, is $x = -b/a$). Because a in the standard equation corresponds to h in Einstein's photoelectric equation, the gradient of the graph of $(E_k)_{max}$ plotted against f is Planck's constant h. Similarly, the intercept on the f-axis is $-(-\phi/h) = +\phi/h$.

COMPTON EFFECT

PHOTON

Einstein is undoubtedly best known for his pioneering work on relativity. However, when he was awarded the Nobel Prize for physics in 1921, the citation did not refer to his monumental contributions to the theory of relativity; instead it referred specifically to 'his discovery of the law of the photoelectric effect'. And as if to underline the importance of the effect it was mentioned again in the Nobel Prize citation two years later when the American physicist Robert Millikan received the prize partly for carrying out experiments that clearly demonstrated the validity of Einstein's photoelectric equation.

We stressed at the beginning of this Section that, according to the particle model of electromagnetic radiation, the particles should each have both energy *and* momentum. Einstein showed that the energy of radiation is quantized, but it remained to show that each quantum has momentum.

6.3 THE COMPTON EFFECT

In the photoelectric effect, the energy of each quantum of radiation is fully *absorbed* by a single electron. The **Compton effect** is a similar process, in which a radiation quantum is *scattered* by a single electron (Figure 49). This process was first considered theoretically in 1922 by the American physicist Arthur Compton, after whom it was of course named, and by the Dutch physicist Peter Debye.

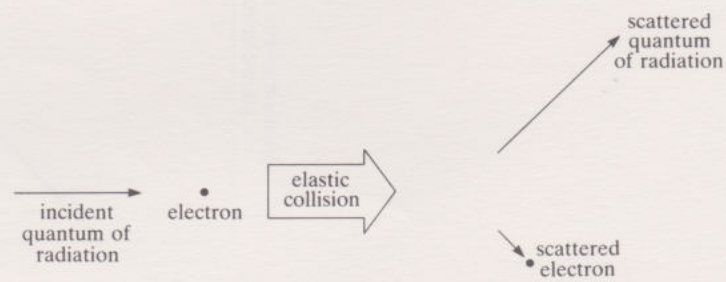


FIGURE 49 The Compton effect.

Compton and Debye analysed how electromagnetic radiation would be scattered by electrons, even before such a process had been observed experimentally. Their analysis assumed that the scattering process can be described by a particle model, in which each radiation quantum has both energy ($E = hf$; Equation 4) and momentum. For reasons based on Einstein's special theory of relativity, Compton and Debye assumed that the magnitude p of the momentum of each quantum is given by

$$p = E/c \tag{7}$$

where E is the energy of the quantum and c the speed of light in a vacuum. The direction of the quantum's momentum was assumed to be the same as the direction of its motion.

ITQ 13 To the nearest order of magnitude, what is the magnitude of the momentum of a quantum of visible light, according to Compton and Debye?

So Compton and Debye pictured the scattering of radiation by electrons in terms of an elastic collision between two particles (Figure 49). What did their analysis show? Well, their calculations showed that the greater the angle through which the radiation quantum is scattered, the greater should be the reduction in its energy and therefore the greater should be the reduction in its associated frequency. Notice that the *direction* of the radiation quantum is important here—this reflects the directional nature of the quantum's momentum. (In contrast, its energy has no associated direction, which is why directions of motion are not important in the photoelectric effect.)

It is important to note that if the Compton scattering process were described by a *wave* model, there should be no reduction in the incident radiation's frequency due to the collision. The wave model predicts that the radiation scattered from an electron has the same frequency as the incident radiation, just as the model predicts that if a blue light beam is shone on a mirror, blue light (i.e. light of the same frequency) will be reflected.

What does experiment say? Is the energy of radiation scattered by an electron less than the energy of the incident radiation (as the particle model predicts), or are the energies the same (as the simple wave model predicts)?

Well, in 1924 Compton showed experimentally, using beams of X-rays, that the theoretical predictions of the particle model were borne out. Just as this model predicted, the energy of the incident X-rays is reduced in the collision; the greater the angle through which a radiation quantum is scattered, the greater is the reduction in its energy. As we have stressed, this cannot be understood in terms of a wave model of radiation—it is almost as if blue light were shone on a target and *red* light were reflected!

Compton's results demonstrated the validity of the particle model of radiation and, in particular, they showed for the first time that each radiation quantum has momentum. The excellent agreement between the theoretical prediction and experiment gave credence to the expression used in the theory for the momentum of the radiation quantum (Equation 7). Later experiments have shown that the Compton effect occurs not only for X-rays, but for all the other types of electromagnetic radiation, including light. Moreover, these experiments confirmed the theoretical description of the effect.

It would be difficult to exaggerate the impact of Compton's work on his colleagues. Compton showed that each radiation quantum has energy *and* momentum. Hence, it was Compton who established beyond doubt the need for a particle model of radiation in accounting for the interaction of radiation with electrons. As Einstein wrote in a popular article for the newspaper *Berliner Tageblatt*,

> The positive result of the Compton experiment proves that radiation behaves as if it consisted of discrete energy projectiles, not only in regard to energy transfer but also in regard to momentum transfer.
>
> (Einstein, 1924)

Partly as a result of Einstein's recommendation, Compton was awarded a Nobel Prize for physics in 1927 'for his discovery of the effect named after him'. The year before, the particles of radiation were first called **photons**—this term is now used universally.

6.4 OTHER APPLICATIONS OF THE PARTICLE MODEL

We have discussed the photoelectric and Compton effects in considerable detail, but there are also other effects that can be understood only by using a photon model of radiation.

There is a whole series of phenomena, perhaps best described under the heading photochemistry, that also require a particle-model explanation. For instance, the absorption of light by photographic film in which the film's crystals of silver bromide are converted to silver, or the detection of light by the retina of the eye, or the process of photosynthesis in plants (Unit 23), are all examples of the interaction of light with matter that are best understood by treating the light as though it were composed of particles of energy.

There is also the problem of explaining how light is emitted by matter, which is one of the most important topics dealt with in Units 11–12. As you might now guess, the key to understanding the processes involved lies in treating the light as photons—as particles.

SUMMARY OF SECTION 6

1 A photon is a particle of electromagnetic radiation.

2 In the photoelectric effect, electrons are ejected from a solid by incident electromagnetic radiation. The results of experiments on the effect cannot be understood by using a simple wave model, but Einstein explained them by assuming that the energy of electromagnetic radiation is quantized, and that each photoelectron is ejected as a result of absorbing all of the energy of a single radiation quantum.

3 In the Compton effect, electromagnetic radiation is scattered by an electron. The results of experiments on the effect cannot be understood by using a simple wave model, but they can be understood if the scattering is modelled as a collision between a single electron and a single radiation quantum that has both energy and momentum.

4 Experiments on the photoelectric effect and the Compton effect have established that:

(a) the energy E of a photon of radiation with frequency f is given by $E = hf$, where h is Planck's constant;

(b) each photon has a momentum of magnitude $p = E/c$, and with a direction that is the same as the direction of the radiation beam.

SAQ 11 Can visible light eject electrons from a piece of copper, which has a work function of 4.4 eV?

SAQ 12 The yellow light from a sodium street lamp has a frequency of 5.09×10^{14} Hz.

(a) What is the energy of each photon of the light, in units of electronvolts?

(b) What is the momentum of each photon of light?

SAQ 13 Which *two* of the following statements are correct?

(a) The results of all experiments on the photoelectric effect can be understood using a simple wave model of electromagnetic radiation.

(b) According to Einstein's theory of the photoelectric effect, an electron ejected from a solid has the same energy as the photon that ejected it.

(c) Einstein's theory of the photoelectric effect assumes that each quantum of radiation has momentum.

(d) Experiments on the Compton effect show that each quantum of radiation has momentum.

(e) In the Compton effect, a photon is scattered by an electron.

7 LIVING WITH THE WAVE–PARTICLE DUALITY

So you might now think that there is a dilemma. What *is* light—waves or particles? The observed diffraction effects argue that it is a wave. Yet the Compton effect argues equally powerfully that it is particles. How can we possibly reconcile these two viewpoints? The answer is, we don't have to—*both* viewpoints are correct!

Now, if this sounds a bit far-fetched to you, then you're in good company—or more accurately, you would have been in good company 80 or 90 years ago—because physicists at the turn of the century also found such an idea hard to swallow. It's probably fair to say that developments in 20th century physics have forced us to look more critically at exactly what science can and cannot do. And it has become increasingly clear that science does not have very much to say in answer to questions beginning 'What is . . .?' Thus the answer to the question 'What is light?' is quite easy: light is light is light . . . . The sort of questions science *can* answer are 'How does light behave?' or 'How do we explain this particular behaviour of light?'. Now, of course, we can begin to answer such questions with:

- When light is diffracted by a double-slit, it behaves like a wave of oscillating electric and magnetic fields.
- When light interacts with the electrons in matter, it behaves like particles, each particle having energy and momentum.

What we are doing in both these answers is *modelling* the behaviour of light. Remember we said in Unit 1 that a model is 'an artificial construction invented to represent or to simulate the properties, the behaviour, or the relationship between individual parts of the real entity being studied'. The wave and particle models we used earlier in the present Unit (Figure 50) are artificial constructions that have helped us to visualize, in our mind's eye, how light behaves. Consequently, the models we use are, of necessity, models that have meaning for us *on our macroscopic* (large-scale) *level*. The mention of waves almost certainly conjures up pictures of waves on the surface of water, because these are the waves that are readily visualized; particles, on the other hand, probably bring to mind billiard balls, or ball-bearings, or some other macroscopic objects. In both of these instances, we are visualizing the familiar in the hope that it will give us some insight into what might be not so familiar. It is important to realize that our models are little more than mental crutches to support and further our understanding.

The same is true of purely mathematical models. It would be nonsense to argue that mathematical equations *are* phenomena. They are ways of

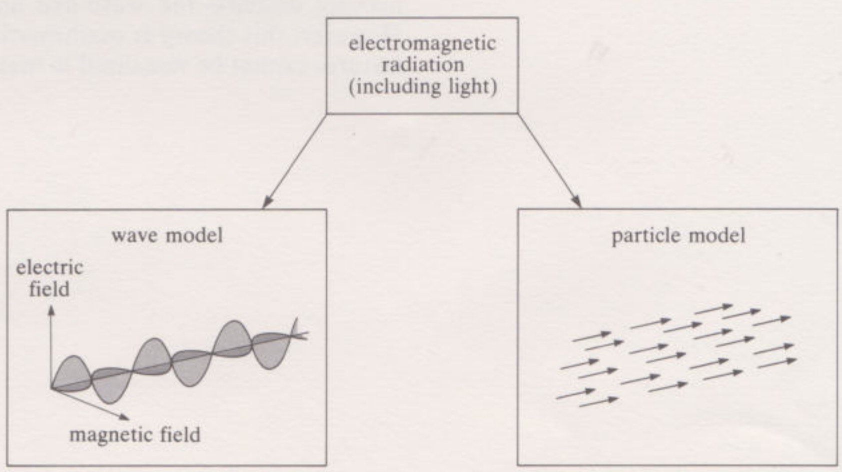


FIGURE 50 The two models of electromagnetic radiation discussed in this Unit: a wave model, in which the radiation is modelled as an electromagnetic wave; and a particle model, in which radiation is modelled as photons.

QUANTUM ELECTRODYNAMICS

describing, understanding, and possibly even increasing our insight into the phenomena: but they are not the phenomena themselves.

So it is with light (and the other types of electromagnetic radiation). There is no macroscopic model available that exhibits *all* the behavioural properties of light. If there were, we would surely use it. The best we can do under the circumstances is use a hybrid of two macroscopic models. In some situations light *behaves* as waves, in others it *behaves* as particles. It is not either. We have to accept a wave–particle *duality*.

So the question now arises: 'How do we know when to use the wave part of the model, and when to use the particle part of the model?' The answer has been illustrated in all our examples throughout this Unit. The wave model enables us to predict the *propagation* behaviour of light—the way in which light travels. The particle model has to be used when explaining how light *interacts with matter*. In any given problem, we may have to make use of both models to reach a complete solution. (There is an example of such a problem in the TV programme 'Light—in search of a model'.) But be warned: *never try to mix the two parts of the dual model together*. To ask what path a photon follows after it has been diffracted by a double slit, or by what mechanism a light wave is converted into a photon before it is absorbed, will serve no useful purpose: such questions are essentially meaningless. Hard though this may be to accept, this is exactly what living with wave–particle duality is all about.

You may well be wondering whether there exists a theory that can simultaneously describe the wave and particle properties of electromagnetic radiation. Well, such a theory does exist—it is called **quantum electrodynamics**, and it was formulated in 1949, independently, by Richard Feynman, Julian Schwinger and Sin-Itiro Tomonaga. This one theory successfully describes the behaviour of radiation in all circumstances. We cannot present this theory here, however, as the mathematics involved is far beyond the level of this Course and many of its key features cannot be visualized in macroscopic terms.

SUMMARY OF SECTION 7

1 In this Unit, we have modelled the behaviour of light (and other types of electromagnetic radiation) using a wave model and a particle model. The models allow us to visualize the behaviour of light in macroscopic terms.

2 The wave model should be used to describe the propagation of light, and the particle model to describe the interaction of light with matter: each model has its own domain of applicability.

3 There exists a theory (quantum electrodynamics) that can simultaneously describe the wave-like and the particle-like behaviour of light. However, this theory is mathematically very complex and many of its key features cannot be visualized in macroscopic terms.

8 TV NOTES: LIGHT—IN SEARCH OF A MODEL

In this programme, we discuss the wave and particle models of light and consider in detail the wave–particle duality of light. We also show that, with the aid of these two models, two practical applications of light, namely solar cells and lens testing, can be understood.

The demonstration of the photoelectric effect, with which we start the programme, is quite dramatic. When ultraviolet radiation is shone onto a piece of zinc placed on a gold-leaf electroscope that has been charged negatively, the zinc rapidly loses its excess electrons. On the other hand, a much more intense beam of red light from a laser apparently has no effect. This observation is explained by the photon model, which assumes that the energy of a photon is proportional to the frequency of the light. Since the frequency of ultraviolet radiation is about twice that of red light, an ultraviolet photon transfers enough energy to an electron to allow it to escape from the zinc, but a red photon transfers insufficient energy for this to happen.

In the programme we do not mention that we took a number of precautions to make sure that the demonstration worked properly. The zinc had to be cleaned (with wire wool) to ensure that pure zinc was exposed to the light, rather than zinc oxide which has a higher threshold frequency. Also, the zinc and the electroscope must first be *negatively* charged; that is, they must have an excess of electrons.

□ Why would the experiment *not* work if the zinc initially had a positive charge?

■ After the electrons had been knocked out of the zinc, they would be pulled back by the electrostatic force. Remember, opposite charges attract, so the positively charged zinc would attract the electrons.

It is also worth pointing out that the electroscope does not discharge unless the zinc is present. This indicates that the threshold frequency of the brass plate of the electroscope is *higher* than the frequency of the ultraviolet radiation from the mercury lamp.

To make maximum use of the energy potentially available in sunlight, it is necessary to use devices (solar cells) made from materials such as silicon that have much *lower* threshold frequencies. The silicon solar cells that we demonstrate in the programme work even when they are illuminated with red light alone.

Whereas the operation of solar cells is easier to explain in terms of a photon model, the important phenomena of diffraction and superposition (interference) are much simpler to visualize in terms of waves. We demonstrate a ripple tank in operation, and use it to show ripples being diffracted by two slits in a barrier. The behaviour of the ripples is essentially the same as the behaviour of light diffracted by two slits, and we explain it in terms of the constructive and destructive superposition of the waves passing through the two slits. We also show that constructive and destructive superposition occur when two pieces of glass in close contact are illuminated. In this case, the bright and dark regions are produced by the superposition of light reflected from two surfaces of the glass (Figure 51). The

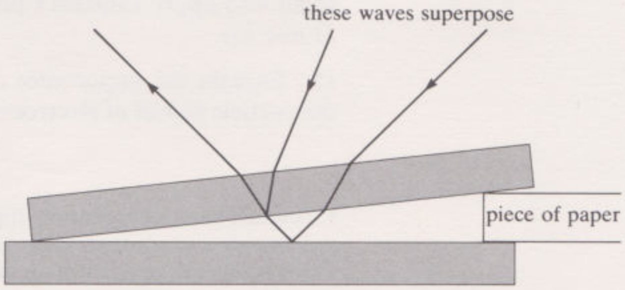


FIGURE 51 The superposition of light from two glass surfaces.

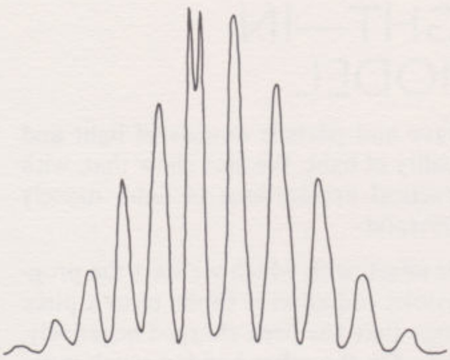


FIGURE 52 The intensity of a diffraction pattern, as recorded by a photomultiplier. (The slight splitting of the left-hand central peak was caused by an instrumental defect—ignore it!)

spacing of the adjacent bright regions decreases when the angle between the two glass surfaces increases, and we show how this phenomenon is utilized in testing lenses.

At the end of the programme, we show both the wave and the particle nature of light in the same experiment. A laser beam is diffracted by a double slit, and we observe the pattern of bright and dark regions that is characteristic of wave-like behaviour. We use a photomultiplier* to measure the number of photons per second arriving at each point in the diffraction pattern, and the graph we plot is reproduced in Figure 52. This graph shows essentially how the intensity varies across the pattern. In order to observe (and hear!) the effects of individual photons, we reduce the intensity of the light. Then, for each photon detected, a spot appears on an oscilloscope and a loudspeaker produces a simultaneous click. What is more, as the photomultiplier is moved, we find alternating regions where many photons are detected and where few photons are detected, corresponding to the bright and dark regions in Figure 52. Thus, even when we are detecting individual photons, the distribution of photons in the diffraction pattern is just as predicted by the wave model!

OBJECTIVES FOR UNIT 10

After you have worked through this Unit, you should be able to:

1 Explain the meaning of, and use correctly, all the terms flagged in the text.

2 Describe and interpret the diffraction patterns observed using a single-slit, a double-slit and a diffraction grating. (*ITQs 1–4*)

3 Recall and be able to apply the relationships $v = f\lambda$ and $f = 1/T$, and be able to relate these parameters to graphical representations of waves. (*AV sequence; SAQs 1–3, 8 and 9*)

4 Show how the principle of superposition accounts for the destructive and constructive superposition of two waves. (*ITQ 5; SAQ 3*)

5 Recall that the wavelength of red light is longer than the wavelength of violet light, and appreciate the consequences in diffraction experiments. (*ITQ 8*)

6 Recall and be able to apply the diffraction equation $d \sin \theta_n = n\lambda$, which specifies the angles at which constructive superposition occurs with both a double-slit and a diffraction grating. (*ITQs 4–7, 9 and 14; SAQs 4–7*)

7 Sketch and interpret an electromagnetic wave of a given wavelength. (*SAQs 8 and 9*)

8 Interpret the electromagnetic spectrum. (*ITQ 10; SAQs 8–10*)

9 Recall and apply the formulae for the energy and momentum of a photon of electromagnetic radiation that has a single frequency. (*ITQs 11 and 13; SAQ 12*)

10 Describe and apply Einstein's theory of the photoelectric effect, and recall and apply Einstein's photoelectric equation. (*ITQs 10 and 12; SAQs 11 and 13*)

11 Explain the importance of the Compton effect in the establishment of the particle model of electromagnetic radiation. (*SAQ 13*)

* You can think of a photomultiplier (and its associated electronic circuits) as being a device that produces a short pulse of electric current for each photon it detects. These pulses can be counted, or displayed in various ways.

ITQ ANSWERS AND COMMENTS

ITQ 1 (a) Figure 53 illustrates the kind of pattern you should have observed when looking through the widest single-slit. As you can see, there is a central region of brightness, flanked symmetrically on either side by a dark region, then a bright region, then another dark region, and so on. If you didn't see this, try the experiment again.

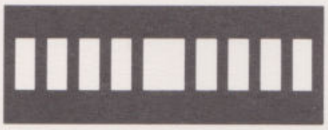


FIGURE 53 Schematic diagram of the diffraction pattern produced by the widest single-slit on the transparency (see ITQ 1).

(b) The light at the centre of the pattern is more or less the 'white' colour of the filament. However, the light away from the centre appears to be split up into little rainbow spectra, with bluish light on the inside (nearest the centre) of the spectrum, and reddish light on the outside. Don't worry if this is not very clear here; it will be much clearer for the double-slits and the grating.

(c) When looking through the (narrower) single-slit in the middle, the pattern of darkness and light appears to be more spread out—there is a bigger distance between the centre of the pattern and the first region of darkness on either side.

(d) Yes. With the narrowest slit, the pattern is spread out even more—so much so, in fact, that the first dark region is now almost on the very extreme of the visible pattern.

ITQ 2 (a) Once again, the light is 'bent'—diffracted—as it passes through the double-slit. Figure 54 shows the kind of pattern that you should have observed. The regions of darkness and brightness are now far more numerous than in the single-slit case. They are also, as far as it is possible to tell by eye, all equally spaced and

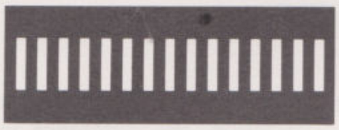


FIGURE 54 Schematic diagram of the diffraction pattern produced by a double-slit on the transparency (see ITQ 2). Each dark line here represents a region of comparative darkness.

of the same width. (Compare this with the single-slit case, in which the central bright region is apparently about double the width of the flanking bright regions.)

(b) Again, the light at the very centre of the pattern is white, but away from the centre it gets broken down into repeated, rainbow-like, spectra.

Within each rainbow spectrum, the bluish light is on the inside (nearest the straight-through direction) and the red on the outside.

(c) As the separation between the slits is *reduced* (going first to the central double-slit, and then to the leftmost double-slit), the repeat-spacing of the dark–bright pattern gets *wider*. The areas of brightness are farther apart with the (closely-spaced) left-hand double-slit, than with the (widely-spaced) right-hand double-slit.

ITQ 3 (a) The pattern observed with the diffraction grating is illustrated in Figure 55. In this pattern, and particularly at the centre, the separation between the regions of brightness is very well defined, i.e. there is a considerable stretch of darkness between adjacent bright regions.

FIGURE 55 Schematic diagram of the diffraction pattern produced by the grating on the transparency (see ITQ 3).

(b) As before, the central area of brightness is white, but it is now quite clear that *all* the other bright areas are spread out into rainbow spectra. Again, the blue light is on the inside, and the red on the outside, of each spectrum. *Within each spectrum, red is bent most, blue least.* Towards the edges of the complete pattern, the spectra are beginning to overlap each other. (By the way, the diffraction grating on the transparency used in the experiment has 40 lines per millimetre.)

ITQ 4 The most obvious answer here is the correct one! The light is spread out *horizontally* by the vertical slit. Hence, the spreading of the light from a *narrow* vertical source can be seen even when the spreading is quite slight (provided that it's more than the approximate *width* of the strip-filament). Had the filament been horizontal (and the slit or slits been vertical), there would have been enormous overlapping of the bright regions horizontally. They would all have blurred into each other, and the spreading of the light would not have been at all clear. Similarly, if the slit(s) had been horizontal and the filament vertical, there would have been vertical overlapping of the bright regions. You can try this yourself if you like.

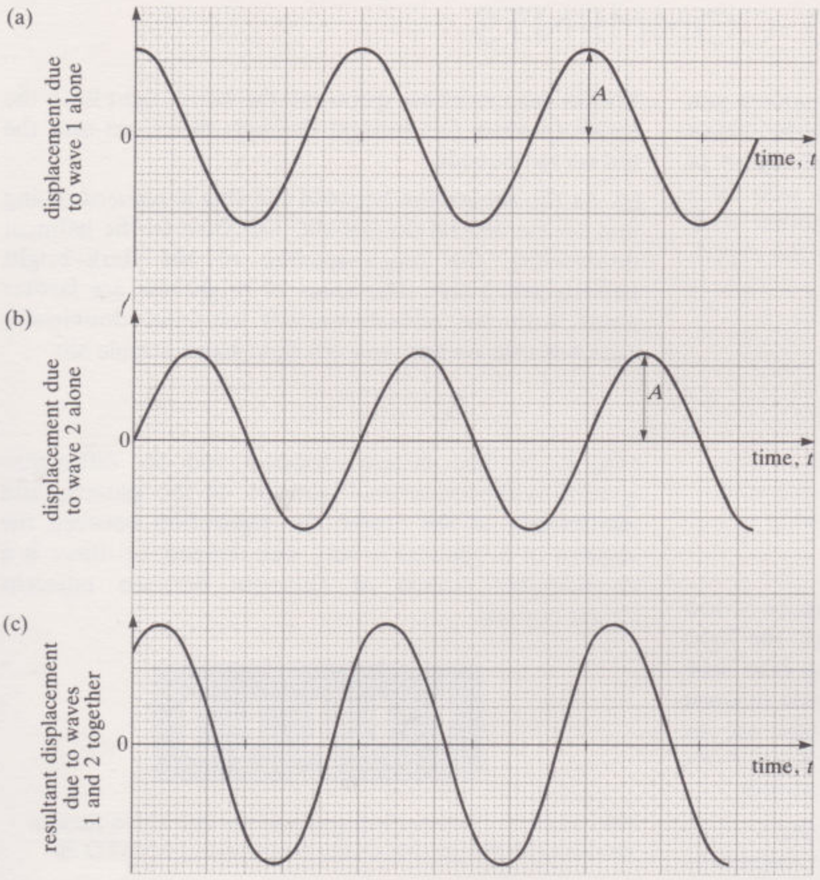


FIGURE 56 For use with the answer to ITQ 5.

ITQ 5 The completed diagram is shown in Figure 56. Note that at $t = 0$, wave 2 has zero displacement (from the average position), whereas wave 1 has a maximum displacement, i.e. waves 1 and 2 are out of step by one-quarter of a complete wave cycle. This addition of the two waves demonstrates that the resultant amplitude at R is *greater* than A (the amplitude of wave 1 and wave 2), but less than $2A$ (the resultant amplitude at point P).

ITQ 6 (a) 560 nanometres.

The diffraction equation is $d \sin \theta_n = n\lambda$ (Equation 2), i.e. $\lambda = (d \sin \theta_n)/n$. For the *first* region of brightness next to the straight-ahead direction, $n = 1$ and $\theta_1 = 0.40°$. Since $d = 0.08$ mm, we have

$$\lambda = \frac{d \sin \theta_1}{1}$$
$$= (0.08\,\text{mm}) \times (\sin 0.40°)$$
$$= (0.08\,\text{mm}) \times 0.0070$$
$$= 5.6 \times 10^{-4}\,\text{mm}$$
$$= 5.6 \times 10^{-7}\,\text{m}$$
$$= 560\,\text{nm}$$

(remembering that $1\,\text{nm} = 10^{-9}\,\text{m}$).

Similarly, for the *second* region of brightness, $n = 2$, and $\theta_2 = 0.80°$.

Hence,

$$\lambda = \frac{d \sin \theta_2}{2}$$
$$= \frac{(0.08\,\text{mm}) \times 0.0140}{2}$$
$$= 5.6 \times 10^{-7}\,\text{m}$$
$$= 560\,\text{nm}$$

As is to be expected, the value of the wavelength is the same in both cases.

(b) 1 120 nm.

For the *second* region of brightness to the right of the straight-ahead direction, the difference in path length must be *two* wavelengths. Therefore

$$\text{path difference} = 2\lambda = 2 \times (560\,\text{nm}) = 1\,120\,\text{nm}$$

ITQ 7 0°, i.e. in the straight-ahead direction itself.

Second order diffraction occurs at an angle where the path difference between waves from adjacent slits is 2λ; first order diffraction occurs when that path difference is 1λ. By extension, zeroth order diffraction should occur when there is zero path difference in the straight-ahead direction, i.e. when θ_0 = zero degrees.

If you prefer the mathematical approach:

$$d \sin \theta_n = n\lambda \qquad \text{in the general case}$$

In the present case, $n = 0$ (zeroth order), and thus $n \times \lambda = 0$. Hence

$$d \sin \theta_0 = 0$$

so $$\sin \theta_0 = 0$$

and $$\theta_0 = 0°$$

(since $\sin 0° = 0$).

ITQ 8 Red light.

Because $\sin \theta_1 \propto \lambda$ (for fixed d), longer wavelengths are bent through larger angles. Hence red is bent most, violet least.

ITQ 9 You can see mathematically why the zeroth order is not spread out. As you discovered in the answer to ITQ 7, in the straight-ahead direction $\theta_0 = 0°$ *whatever* the value of λ. Hence, the straight-ahead line will be the same colour as the incident light source.

ITQ 10 Type (d) because, of the types of radiation used, only X-rays have frequencies above 1.1×10^{15} Hz, which is the frequency below which no electrons are emitted from iron, as you saw in the text.

ITQ 11 (a) (i) 10^{-19} J; (ii) 1 eV.

(i) The frequency of visible light is approximately 5×10^{14} Hz, so (using Equation 4, $E = hf$) the energy E of a quantum of visible light is given by:

$$E = (6.63 \times 10^{-34}\,\text{J s}) \times (5 \times 10^{14}\,\text{Hz})$$

$$\approx 3.3 \times 10^{-19}\,\text{J}$$

$$= 10^{-19}\,\text{J} \quad \text{to the nearest order of magnitude (i.e. to the nearest power of ten)}$$

(ii) Because $1\,\text{eV} \approx 1.6 \times 10^{-19}$ J, the energy of a quantum of visible light is

$$E \approx \frac{3.3 \times 10^{-19}}{1.6 \times 10^{-19}}\,\text{eV}$$

$$\approx 2\,\text{eV}$$

$$= 1\,\text{eV} \quad \text{to the nearest order of magnitude}$$

(b) 10^{19} quanta.

From part (a)(i), the energy of each quantum of visible light is 3.3×10^{-19} J. Therefore the number of quanta of visible light emitted by the bulb in one second is

$$\frac{10\,\text{J}}{3.3 \times 10^{-19}\,\text{J}}$$

$$= 3 \times 10^{19}$$

$$= 10^{19} \quad \text{to the nearest order of magnitude}$$

Note how large this number is—the light bulb emits about 10 000 000 000 000 000 000 quanta of visible light in one second!

ITQ 12 (a) Electrons will be ejected with a range of kinetic energies up to a maximum of 6 eV.

According to Einstein's photoelectric equation (Equation 6):

$$(E_k)_{max} = 10\,\text{eV} - 4\,\text{eV} = 6\,\text{eV}$$

(b) No, because the energy of each quantum is not sufficient to eject electrons.

For electrons to be ejected, the energy of the quantum must be greater than 4 eV, i.e. greater than the work function of lead. More formally, you can see from Equation 6 that if $(E_k)_{max}$ is to be greater than zero, the energy hf of the quantum must be greater than the work function ϕ of the solid.

ITQ 13 $10^{-27}\,\text{kg m s}^{-1}$.

From ITQ 11, the energy of a quantum of visible light is approximately 3.3×10^{-19} J. Hence, the magnitude of the momentum of a quantum of visible light is, according to Equation 7,

$$\frac{3.3 \times 10^{-19}\,\text{J}}{3.0 \times 10^{8}\,\text{m s}^{-1}}$$

$$= 1.1 \times 10^{-27}\,\text{kg m s}^{-1}$$

$$= 10^{-27}\,\text{kg m s}^{-1} \quad \text{to the nearest order of magnitude}$$

SAQ ANSWERS AND COMMENTS

SAQ 1 (a) The wave's amplitude is 20 mm; it is the distance from the average level to the maximum displacement level.

(b) The period is 0.4 seconds; it is the time interval from (say) one peak to the next in the displacement–time graph.

(c) The frequency f is 2.5 Hz; it is calculated as follows:

$$\begin{aligned} f &= 1/T \\ &= 1/(0.4\,\mathrm{s}) \quad \text{from (b)} \\ &= 2.5\,\mathrm{s}^{-1} \text{ or } 2.5\,\mathrm{Hz} \end{aligned}$$

(d) The wavelength λ is 2 m; it cannot be read from the graph. It can be found from:

$$v = f\lambda$$

Dividing both sides of this equation by f and rearranging:

$$\lambda = v/f = (5.0\,\mathrm{m\,s^{-1}})/(2.5\,\mathrm{s^{-1}}) = 2\,\mathrm{m}$$

SAQ 2 $v_1 = f_1\lambda_1 = (0.30\,\mathrm{s^{-1}}) \times (17\,\mathrm{m}) = 5.1\,\mathrm{m\,s^{-1}}$.

$v_2 = f_2\lambda_2 = (0.90\,\mathrm{s^{-1}}) \times (2.0\,\mathrm{m}) = 1.8\,\mathrm{m\,s^{-1}}$.

The speeds are quite different. Because travel time = distance/speed, the longer wavelength waves (a swell) will reach the shore, 100 km away, approximately 10 hours before the short wavelength waves (a choppy sea).

SAQ 3 The resultant displacement is shown in Figure 57. The dots on the resultant wave (bold curve) show the points where addition of the two waves is particularly easy.

SAQ 4 (a) $\lambda = 500\,\mathrm{nm}$.

The diffraction equation for constructive superposition is:

$$d \sin\theta_n = n\lambda$$

Here $n = 2$ (second order diffraction), and $d = 2 \times 10^{-6}\,\mathrm{m}$. Also, since $\theta_2 = 30°$, $\sin\theta_2 = 0.5$. Hence:

$$(2 \times 10^{-6}\,\mathrm{m}) \times 0.5 = 2 \times \lambda$$

Therefore, dividing both sides by 2, the wavelength λ of the laser light is given by:

$$\lambda = 0.5 \times 10^{-6}\,\mathrm{m} = 5 \times 10^{-7}\,\mathrm{m} = 500\,\mathrm{nm}$$

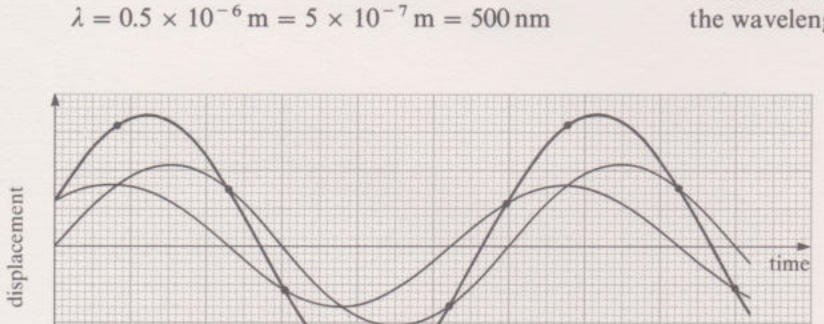


FIGURE 57 For use with the answer to SAQ 3.

(b) $\theta_1 \approx 14°$.

Using the diffraction equation,

$$\begin{aligned} \sin\theta_1 &= \lambda/d \\ &= \frac{5 \times 10^{-7}\,\mathrm{m}}{2 \times 10^{-6}\,\mathrm{m}} \\ &= 0.25 \end{aligned}$$

As you should be able to show using your calculator, the angle whose sine is 0.25 is 14° (to two significant figures).

SAQ 5 If you've decided that this question cannot be answered without knowing d for the grating, then you've jumped to a hasty, and incorrect, conclusion! Have another go at the problem before reading on.

For second-order diffraction, the path differences between light from adjacent slits is two wavelengths; for third-order diffraction, the path difference is three wavelengths. If the orange light and violet light are diffracted through the same angle, then the two path differences must be identical. That is:

$$3 \times \lambda_{\text{violet}} = 2 \times \lambda_{\text{orange}} = 2 \times (600\,\mathrm{nm}) = 1\,200\,\mathrm{nm}$$

$$\lambda_{\text{violet}} = \frac{1\,200\,\mathrm{nm}}{3} = 400\,\mathrm{nm}$$

If you prefer to work with the diffraction equation, then

$$d \sin(\theta_2)_{\text{orange}} = 2\lambda_{\text{orange}}$$

and $d \sin(\theta_3)_{\text{violet}} = 3\lambda_{\text{violet}}$

But $\sin(\theta_2)_{\text{orange}} = \sin(\theta_3)_{\text{violet}}$, because the diffraction angle is the same in the two cases. Hence, $2\lambda_{\text{orange}} = 3\lambda_{\text{violet}}$, as before. This again gives $\lambda_{\text{violet}} = 400\,\mathrm{nm}$.

SAQ 6 Superposition is constructive at P_1, and destructive at P_2.

Constructive superposition occurs when the path difference from the two slits is a whole number of wavelengths; destructive superposition occurs when the path difference is an odd number of half-wavelengths. Here, the wavelength is 10 mm.

At P_1:

$$82\,\text{mm} - 72\,\text{mm} = 10\,\text{mm} = \text{one wavelength}$$

There will be constructive superposition at this point.

At P_2:

$$93\,\text{mm} - 88\,\text{mm} = 5\,\text{mm} = \text{half a wavelength}$$

There will be destructive superposition at this point.

Note that the slit separation is not required, to answer this question.

SAQ 7 At 0° (straight-through position), and at approximately 17°, 35° and 59° to either side of the straight-through position.

You must use the equation $d \sin \theta_n = n\lambda$. For this problem, $d = 35\,\text{mm}$ and $\lambda = 10\,\text{mm}$. Constructive superposition will occur for values of n equal to 0, 1, 2, 3, etc. Rearranging the equation,

$$\sin \theta_n = \frac{n\lambda}{d} = \frac{n \times 10}{35} = \frac{n}{3.5}$$

We now work out the various values of $\sin \theta_n$ using this equation, and then use a calculator to find the values of θ_n to which they correspond.

When $n = 0$, $\quad \sin \theta_0 = 0$

giving $\quad \theta_0 = 0°$

When $n = 1$, $\quad \sin \theta_1 = \frac{1}{3.5} \approx 0.29$

giving $\quad \theta_1 \approx 17°$

When $n = 2$, $\quad \sin \theta_2 = \frac{2}{3.5} \approx 0.57$

giving $\quad \theta_2 \approx 35°$

When $n = 3$, $\quad \sin \theta_3 = \frac{3}{3.5} \approx 0.86$

giving $\quad \theta_3 \approx 59°$

When $n = 4$, $\quad \sin \theta_4 \approx \frac{4}{3.5} = 1.14$

No value can be found for θ_4, since $\sin \theta_4$ turns out to be greater than 1, which is not possible. This applies to all values of θ_n where n is greater than 3, for this particular double-slit and wavelength.

So, in addition to the straight-ahead 'beam', there will be three 'beams' to the left and three to the right of the straight-ahead direction.

SAQ 8 $1.4 \times 10^4\,\text{m}$.

The units of the frequency given in the question are kilohertz, kHz, so it is first necessary to express the quantity in SI units of hertz: $22\,\text{kHz} = 22 \times 10^3\,\text{Hz} = 2.2 \times 10^4\,\text{s}^{-1}$ (remember $1\,\text{Hz} = 1\,\text{s}^{-1}$). Now Equation 3 ($c = f\lambda$) can be rearranged and used to find the wavelength λ of the waves:

$$\begin{aligned}\lambda &= c/f \\ &= \frac{3.0 \times 10^8\,\text{m}\,\text{s}^{-1}}{2.2 \times 10^4\,\text{s}^{-1}} \\ &\approx 1.4 \times 10^4\,\text{m}\end{aligned}$$

SAQ 9 (a) $1.2 \times 10^{-1}\,\text{m}$.

Using Equation 3,

$$\begin{aligned}\lambda &= \frac{3.00 \times 10^8\,\text{m}\,\text{s}^{-1}}{2.45 \times 10^9\,\text{Hz}} \\ &\approx 1.2 \times 10^{-1}\,\text{m}\ (=12\,\text{cm})\end{aligned}$$

(b) Your diagram should resemble the electromagnetic wave shown in Figure 58. Note that the wavelength of the wave is approximately 12 cm, as you saw in part (a).

SAQ 10 No, the radiation is not visible. As you can see from the electromagnetic spectrum in Figure 40 (or on the back cover) electromagnetic radiation with a wavelength of approximately 800 nm is in the infrared region of the spectrum.

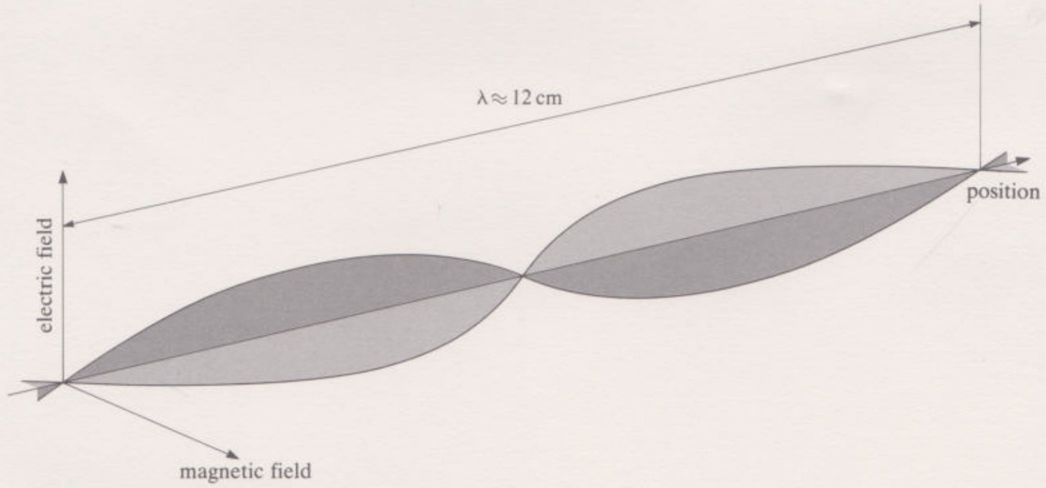


FIGURE 58 For use with the answer to part (b) of SAQ 9.

SAQ 11 No, because photons of visible light do not have sufficiently high energy.

For a photon to eject an electron from copper, it must have an energy greater than 4.4 eV, the work function of copper. The most energetic (highest frequency) form of visible light is violet light, which has a maximum frequency of 7.5×10^{14} Hz. The energy of a photon of this light is given by:

$$E = hf$$
$$= (6.63 \times 10^{-34}\,\mathrm{J\,s}) \times (7.5 \times 10^{14}\,\mathrm{Hz})$$
$$= 5 \times 10^{-19}\,\mathrm{J} \approx 3\,\mathrm{eV}$$

(using the conversion factor $1\,\mathrm{eV} \approx 1.602 \times 10^{-19}\,\mathrm{J}$ given on the back cover). This is less than 4.4 eV, the work function of copper.

SAQ 12 (a) 2.1 eV.

Using Equation 4, the energy of the photon of yellow light is given by:

$$E = hf$$
$$= (6.63 \times 10^{-34}\,\mathrm{J\,s}) \times (5.09 \times 10^{14}\,\mathrm{Hz})$$
$$= 3.37 \times 10^{-19}\,\mathrm{J}$$
$$= 2.1\,\mathrm{eV}$$

(b) $1.1 \times 10^{-27}\,\mathrm{kg\,m\,s^{-1}}$ in the direction in which the photon is travelling.

Using Equation 7 ($p = E/c$), the magnitude p of each photon's momentum is:

$$p = \frac{3.37 \times 10^{-19}\,\mathrm{J}}{3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}} = 1.1 \times 10^{-27}\,\mathrm{kg\,m\,s^{-1}}$$

Note that, in order to obtain the magnitude of the momentum in the SI units of $\mathrm{kg\,m\,s^{-1}}$, it is essential to use values of E and c that are also in SI units, when substituting into Equation 7. You cannot use the value of E expressed in units of electronvolts!

SAQ 13 Statements (d) and (e) are correct; see Section 6.3.

Statement (a) is incorrect, see Section 6.2.2; (b) is incorrect: the energy of the ejected electron is equal to the energy of the incident photon minus the energy required to eject the electron from the solid; (c) is incorrect: it is necessary to take into account only the energy of the incident radiation.

ACKNOWLEDGEMENTS

The Course Team would like to thank David Barfoot for his help in producing the computer-generated diagrams used in this Unit.

Grateful acknowledgement is made to the following sources for permission to reproduce Figures in this Unit:

Figure 7 Coastal Engineering Research Center, US Dept. of the Army; *Figures 8a, 8b and 29* Education Development Center, Newton, Mass.; *Figure 25b Project Physics Course*, 1970, Rinehart and Winston; *Figure 36a* G.R. Graham, Cambridgeshire College of Arts and Technology.

INDEX FOR UNIT 10